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MULTIDIMENSIONAL AND FUZZY INDICATORS DEVELOPMENTS

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1. The multidimensional and fuzzy set approach to poverty measurement

1.1 Traditional poverty approach

The traditional poverty approach is characterized by a simple dichotomization of the population into poor and non poor defined in relation to some chosen poverty line that represents a certain percentage (generally 50%, 60% or 70%) of the mean or the median of the equivalised income[1] distribution.

The traditional poverty method takes place in two different and successive stages: the first aims to identify who is poor and who is not according to whether a person's income is below a critical threshold, the poverty line; the second stage consists of summarising the amount of poverty in aggregate indices that are defined in relation to the income of the poor and the poverty line.

This approach presents two main limitations: firstly, it is unidimensional, i.e. it refers to only one proxy of poverty, namely low income or consumption expenditure, and secondly it divides the population into a simple dichotomy.

However, poverty is a complex phenomenon that cannot be reduced solely to monetary dimension but it must also take account of non–monetary indicators of living conditions; moreover it is not an attribute that characterises an individual in terms of presence or absence, but is rather a vague predicate that manifests itself in different shades and degrees.

1.2 Fuzzy and multidimensional approach

Nowadays the multidimensional nature of poverty is a widely recognised fact, not only by the international scientific community, but also by many official statistical agencies (e.g. Eurostat, Istat) and by international institutions (United Nations, World Bank). This fact implies a more complete and realistic vision of this phenomenon and also an increased complexity at both the conceptual and the analytical levels. Such a complexity determines the need for adequate tools of analysis and the availability of statistical data that have to be adequate too, complete and reliable.

[1] The equivalised income of a household is obtained by dividing its total disposable income by the household's equivalent size computed by using an equivalence scale which takes into account the actual size and composition of the household.

The fuzzy approach considers poverty as a matter of degree rather than an attribute that is simply present or absent for individuals in the population. In this case, two additional aspects have to be introduced:

i. The choice of *membership functions* (m.f.), i.e. quantitative specification of individuals' or households' degrees of poverty and deprivation;
ii. The choice of rules for the manipulation of the resulting fuzzy sets, as complements, intersections, union and aggregation.

1.2.1. Fuzzy monetary

In the conventional approach, the m.f. may be seen as $\mu(y_i) = 1$ if $y_i < z$, $\mu(y_i) = 0$ if $y_i \geq z$ where y_i is the equivalised income of individual *i* and *z* is the poverty line.

An early attempt to incorporate the concept of poverty as a matter of degree at methodological level was made by Cerioli and Zani (1990) who drew inspiration from the theory of *Fuzzy Sets* initiated by Zadeh (1965). They proposed the introduction of a transition zone $(z_1 - z_2)$ between the two states, a zone over which the m.f. declines from 1 to 0 linearly:

$$\mu_i = 1 \; if \; y_i < z_1; \quad \mu_i = \frac{z_2 - y_i}{z_2 - z_1} \; if \; z_1 \leq y_i \leq z_2; \quad \mu_i = 0 \; if \; y_i > z_2 \tag{1.1}$$

Cheli and Lemmi (1995) in their *Totally Fuzzy and Relative* approach attempted to overcome the limits of Cerioli and Zani membership function, that is, the arbitrary choice of the two threshold value and the linear form of the function within such values. They defined the *m.f.* as the distribution function $F(y_i)$ of income, normalized (linearly transformed) so as to equal 1 for the poorest and 0 for the richest person in the population. Formally:

$$\mu_i = (1 - F_i) \tag{1.2}$$

where F_i is the income distribution function. By definition, the mean of this *m.f.* is always 0.5. In order to make this mean equal to some specified value (such as 0.1) so as to facilitate comparison with the conventional poverty rate, Cheli (1995) takes the m.f. as normalized distribution function, raised to some power $\alpha \geq 1$. Formally:

$$\mu_i = FM_i = (1 - F_{(M),i})^{\alpha} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma \mid y_\gamma > y_i}{\sum_{\gamma=2}^{n} w_\gamma \mid y_\gamma > y_1} \right)^{\alpha}, i = 1,2,...,n; \mu_n = 0 \qquad (1.3)$$

where y_i is the equivalised income of the *i*-th individual, $F_{(M),i}$ is the value of the income distribution function $F(y_i)$ for the *i*-th individual, $(1 - F_{(M),i})$ is the proportion of individuals less poor than the person concerned with mean ½ by definition, w_γ is the sample weight of individual of rank γ in the ascending income distribution and α is a parameter.

The value of α is arbitrary, but Cheli and Betti (1999) have chosen the parameter α so that the mean of the m.f. is equal to the head count ratio computed for the official poverty line. Increasing the value of this exponent implies giving more weight to the poorer end of the income distribution.

Betti and Verma (1999) have used a somewhat refined version of the expression (1.3) in order to define what they called Fuzzy Monetary indicator (FM):

$$\mu_i = FM_i = (1 - L_{(M)i})^{\alpha} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma y_\gamma \mid y_\gamma > y_i}{\sum_{\gamma=2}^{n} w_\gamma y_\gamma \mid y_\gamma > y_1} \right)^{\alpha}, i = 1,2,...,n; \mu_n = 0 \qquad (1.4)$$

where y_γ is the equivalised income and $L_{(M),i}$ represent the value of the Lorenz curve of income for individual *i*; then $1 - L_{(M),i}$ represents the share of the total equivalised income received by all individuals who are less poor than the person concerned. It varies from 1 for the poorest to 0 for the richest individual. The mean of $1 - L_{(M),i}$ values equals (1+G)/2, where G is the Gini coefficient of the distribution.

1.2.2. Fuzzy supplementary

In addition to the level of monetary income, the standard of living of households and individuals can be described by a host of indicators, such as housing conditions, possession of durable goods, perception of hardship, expectations, norms and values.

To quantify and put together diverse indicators of deprivation several steps are necessary. Specially, decisions are required to assigning numerical values to the ordered

categories, weighting the score to construct composite indicators, choosing their appropriate distributional form and scaling the resulting measures in a meaningful way.

Firstly, from the large set which may be available, a selection has to be made of indicators which are substantively meaningful and useful for a given analysis. Secondly, it is useful to identify the underlying dimensions and to group the indicators accordingly (these steps will be described in details in the next sections). Whelan *et al.* (2001) suggest, as the first stage in an analysis of life-style deprivation, examining systematically the range of deprivation items to see whether the items cluster into distinct groups. Factor analysis can be used to identify such clusters of interrelated variables.

Moreover, it is necessary to assign numerical values to the ordered categories and to weight and scale measures. Individual items indicating non-monetary deprivation often take the form of simple "yes/no" dichotomies or sometimes ordered polytomies. The simplest scheme for assigning numerical values to categories is by assigning that the ranking of the categories represents an equally-spaced metric variable. Cerioli and Zani (1990) defined the membership function of an individual as follows.

If a vector of *k* categorical variables $X_1,...,X_k$ is observed on the *n* individuals of the population, the membership function of the fuzzy set of the poor can be defined as:

$$\mu_A(i) = \frac{\sum_{j=1}^{k} g(x_{ij}) w_j}{\sum_{j=1}^{k} w_j} \qquad i = 1, \ldots, n \qquad (1.5)$$

where $g(x_{ij}) = 1$ if the corresponding x_{ij} denotes deprivation and $g(x_{ij}) = 0$ otherwise. w_j denotes the weight of the variable X_j ($j = 1, \ldots, k$).

If variable X_j is of ordinal scale, it is possible to identify a modality $x_j^{'}$ of X_j denoting lack of resources and a modality $x_j^{''}$ that excludes poverty. These modalities are put in decreasing order beginning with the one that denotes the greatest deprivation. If $\psi_j^{'}$, $\psi_j^{''}$, ψ_{ij} represent the score of categories $x_j^{'}$, $x_j^{''}$, x_{ij} respectively, then:

$$g(x_{ij}) = \begin{cases} 1 & \text{if } \psi_{ij} \le \psi_j^{'} \\ \dfrac{\psi_j^{'} - \psi_{ij}}{\psi_j^{''} - \psi_j^{'}} & \text{if } \psi_j^{'} \le \psi_{ij} \le \psi_j^{''} \\ 0 & \text{if } \psi_{ij} \ge \psi_j^{''} \end{cases} \qquad (1.6)$$

For the weights w_j, Cerioli and Zani proposed the following specifications:

$$w_j = \ln \frac{1}{p_j} \qquad (1.7)$$

where p_j is the proportion of individuals with deprivation in variable X_j. Substituting (1.7) in (1.6) we obtain:

$$\mu_A(i) = \frac{\sum_{j=1}^{k} g(x_{ij}) \ln \frac{1}{p_j}}{\sum_{j=1}^{k} \ln \frac{1}{p_j}} \qquad (1.8)$$

A collective index of poverty is simply obtained by Cerioli and Zani using the relative cardinality (Dubois and Prade, 1980) of the fuzzy set of the poor: $|A| = \sum_{i=1}^{n} \mu_A(i)$. Such an index, included between 0 and 1, represents the proportion of individuals that belong to the fuzzy subset of the poor and it is given by:

$$P = \frac{|A|}{n} \qquad (1.9)$$

Cheli and Lemmi (1995) proposed an improvement by replacing the simple ranking of the categories with their distribution function in the population. Formally:

$$g(x_{ij}) = H(x_j) \qquad (1.10)$$

where $H(x_j)$ is the sampling distribution function of the variable X_j. The normalised form is given by:

$$g(x_{ij}) = g(x_j^{(k)}) = \begin{cases} 0 & \text{if } x_{ij} = x_j^{(1)}; k = 1 \\ g(x_j^{(k-1)}) + \dfrac{H(x_j^{(k)}) - H(x_j^{(k-1)})}{1 - H(x_j^{(1)})} & \text{if } x_{ij} = x_j^{(k)}; k > 1 \end{cases} \qquad (1.11)$$

where $x_j^{(1)},...,x_j^{(m)}$ represent the categories of the variable X_j arranged in increasing order with respect to the risk poverty and $H(x_j^{(k)})$ is the distribution function of the variable X_j once its categories have been arranged as described above.

In this way, a 0 *m.f.* value is always associated with the modality corresponding to the lowest risk of poverty, whereas value 1 is associated with the modality corresponding to the highest risk. Cheli and Lemmi proposed the following weights:

$$w_j = \ln(1/\overline{g(x_j)}) \qquad (1.12)$$

where $\overline{g(x_j)} = \frac{1}{n}\sum_{i=1}^{n} g(x_{ij})$ represents the fuzzy proportion of the poor with respect to X_j and if X_j is dichotomic it coincides with the crisp proportion p_j.

An early attempt to choose an appropriate weighting system of several indicators at macro level data was made by Ram (1982), using principal components analysis, which was also adopted by Maasoumi and Nickelsburg (1988). At the micro level, Nolan and Whelan (1996) adopted factor analysis. In order also to give more weight to more widespread items, Cerioli and Zani (1990) specified the weights of any item as a function of the proportion deprived of the item. To avoid redundancy in the choice of weights, Betti and Verma (1999) proposed the item weights to comprise two factor: i) the first factor is determined by the variable's dispersion and it may be taken as proportional to the coefficient of variation of deprivation score for the variable concerned; ii) the second factor is taken as a function of the correlation of any item with other items, in such manner that it is not affected by the introduction of variables entirely uncorrelated with the item concerned, but is reduced proportionately to the number of highly correlated variables present.

As in the Fuzzy Monetary approach, the individual's degree of non-monetary deprivation FS_i can be defined in two alternative manners:

i. The proportion of individuals who are less deprived than *i*:

$$\mu_i = FS_i = (1 - F_{(S),i})^{\alpha_s} \qquad (1.13)$$

where $F_{(S),i}$ is the distribution function of *S* evaluated for individual *i*.

ii. The share of the total non-deprivation *S* assigned to all individuals less deprived than *i*:

$$\mu_i = FS_i = (1 - L_{(S),i})^{\alpha_s} \qquad (1.14)$$

where $F_{(S),i}$ is the value of the Lorenz curve of *S* for individual *i*.

The parameter α_s is determined so as to make the overall non-monetary deprivation rate numerically identical to the monetary poverty rate *H*.

1.3 Combination

In the previous sections, we have defined fuzzy measures of poverty and deprivation in multiple dimensions: monetary poverty on the one hand, and non-monetary deprivation in different aspects of life, on the other. The next step of interest in multidimensional analysis is to identify the extent to which deprivation in different dimensions tends to overlap for individual units, households or persons. For this purpose some operations on the fuzzy sets have to be defined.

Let us consider only two dimensions of deprivation, monetary poverty *m*, and non-monetary deprivation *s*. In the conventional, 'crisp' formulation, individuals are categorised as deprived and non-deprived in each of the two dimensions. We can view any individual as belonging to one and only one of the four subpopulations defined by the intersections $m \cap s$ (m, s = 0,1).

Fuzzy set operations are a generalisation of the corresponding 'crisp' set operations in the sense that the former reduce to (exactly reproduce) the latter when the fuzzy membership functions, being in the whole range [0,1], are reduced to a 0,1 dichotomy.

There are, however, more than one ways in which the fuzzy set operations can be formulated, each representing an equally valid generalisation of the corresponding crisp set operations. The choice among alternative formulations has to be made primarily on substantive grounds: some options are more appropriate (meaningful, convenient) than others, depending on the context and objectives of the application. While the rules of fuzzy set operations cannot be discussed fully in this paper, we need to clarify their application specifically for the study of poverty and deprivation.

Since fuzzy sets are completely specified by their membership functions, any operation with them is defined in terms of the membership functions of the original fuzzy sets involved. For simplicity, let be (a, b) the membership functions of two sets for individual *i*, where $a = FM_i$ and $b = FS_i$, $s_1 = \min(a,b)$, $s_2 = \max(a,b)$ and $\bar{a} = 1 - a$, $a \cap b$, $a \cup b$ the basic set operations of complementation, intersection and union.

Table 1.1 displays the most common ways to specify fuzzy intersection and union that satisfy a set of essential requirements such as 'reduction to the crisp set operation', 'boundary condition', 'monotonicity', 'cummutativity', etc. (for details see Klir and Yuan, 1995).

Table 1.1 Basic forms of fuzzy set intersections and unions

	Intersection $a \cap b$	Union $a \cup b$
Standard	$i(a, b) = \min(a, b) = i_{\max}$	$u(a, b) = \max(a, b) = u_{\min}$
Algebraic	$i(a, b) = a * b$	$u(a, b) = a + b - a * b$
Bounded	$i(a, b) = \max(0, a + b - 1)$	$u(a, b) = \min(1, a + b)$

The Standard fuzzy operations provide the largest intersection and by contrast the smallest union among all the permitted forms. They are appropriate for intersection and union of similar fuzzy sets, i.e. sets for which the membership functions are expected to have a substantial positive correlation, but not uniformly throughout in the application to poverty analysis because their sum would exceed 1 and the marginal constraints would not be satisfied. An obvious example is a pair of sets, one defining the degree of income poverty, and the other deprivation of a certain type such as 'basic non monetary deprivation'.

The Bounded operator is appropriate for the aggregation of dissimilar sets for which the membership functions are expected to have a substantial negative correlation. This, for example, will be the case with one set defining the degree of presence of poverty, and the other defining the degree of absence of deprivation in a certain dimension.

The Algebraic operator is appropriate for the aggregation of sets in the absence of such correlations. It is the only one that satisfies the marginal constraints but it could give non acceptable results.

Betti and Verma (2004) proposed to use in the analysis of fuzzy sets defining deprivation in different dimensions the so called 'Composite' set operator:

1. For sets representing similar states – such as the presence or absence of both types of deprivation – the Standard operations (which provide larger intersections than Algebraic operations) are used.

2. For sets representing dissimilar states- such as the presence of one type but the absence of the other type of deprivation – the Bounded operations (which provide smaller intersections than Algebraic operations) are used.

A possible, more flexible, but of course more demanding on data and substantive judgement alternative would be to consider a weighted combination the Composite and Algebraic set operators, for instance in the following form, which also meets the consistency requirement:

1. For sets representing similar states $\rightarrow$ (1-*w*)(Standard) + *w*(Algebraic)
2. For sets representing dissimilar states $\rightarrow$ (1-*w*)(Bounded) + *w*(Algebraic)

Parameter *w* can be thought of as a measure of the degree to which different types of states can be distinguished. When $w = 0$ we have the Composite scheme defined above, with its sharp distinction between similar and dissimilar states. With w = 1, we have the Algebraic scheme, applicable when the different states are 'neutral' with respect to each other. With $0 < w < 1$, one may represent intermediate types of situations.

Table 1.2 shows the application of this Composite set operations and Graph. 1.1 illustrates them graphically.

Table 1.2. Joint measures of deprivation according to the Betti and Verma Composite operation

		Non-monetary deprivation		
		non-poor (0)	poor (1)	Total
Monetary deprivation	non-poor	$\min(1-FM_i, 1-FS_i) = 1-\max(FM_i, FS_i)$	$\max(0, FS_i - FM_i)$	$1-FM_i$
	poor	$\max(0, FM_i - FS_i)$	$\min(FM_i, FS_i)$	FM_i
	Total	$1-FS_i$	FS_i	1

In the Graph 1.1, that shows intersections, the degree of membership in the "universal set" X is represented by a rectangle of unit length and the individual's memberships on the two subset (say, $0 \le a \le 1$, $0 \le b \le 1$ and their complements) have been placed within it. Different forms of fuzzy set operations (Table 1.1) are reproduced by different placements of the subset memberships within the rectangle for X. The Standard form, appropriate for similar sets, is represented by placing the two memberships (a, b) on the

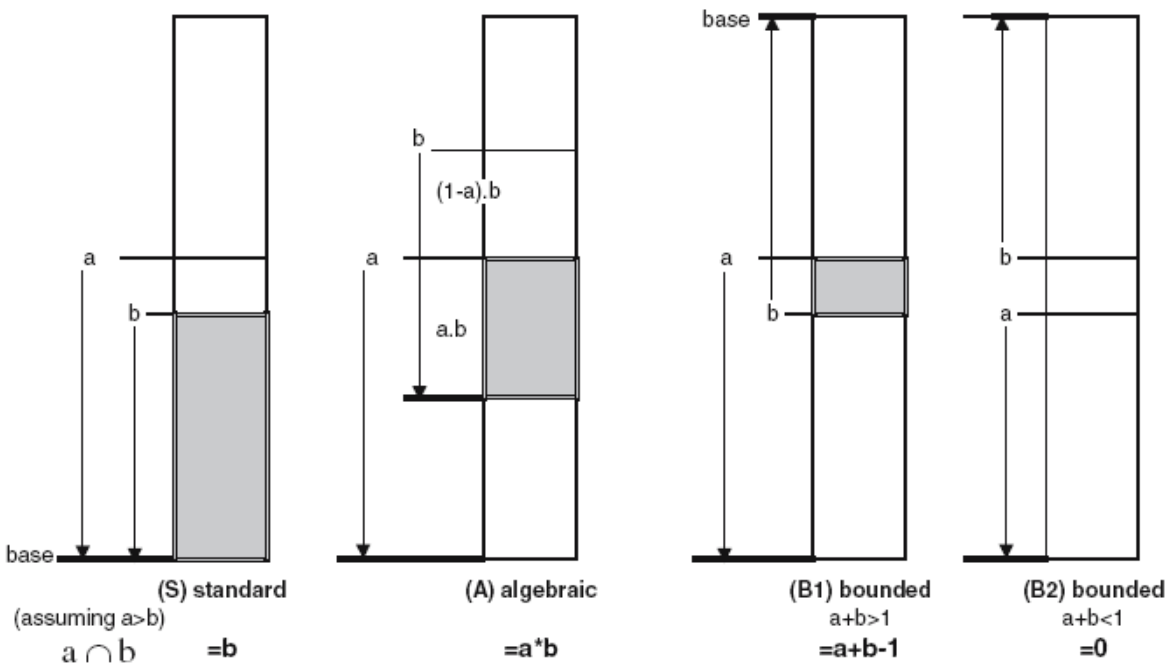
same base, so that their intersection is min(a, b) and union is max(a, b). In the Bounded form, appropriate for dissimilar sets, the two sets are placed et the opposite ends of X, thus their intersection is max(0, a+b-1) and union is min(1, a+b). Similarly, we can represented fuzzy sets unions.

The propensity to income poverty, FM_i, and the overall non-monetary deprivation propensity, FS_i, may be combined to construct composite measures which indicate the extent to which the two aspects of income poverty and non-monetary deprivation overlap for the individual concerned. These measures, at the individual level *i*, are:

i. *Manifest deprivation* (M_i), representing the propensity to both income poverty and non-monetary deprivation simultaneously:

ii. *Latent deprivation* (L_i), representing the individual being subject to at least one of the two, income poverty and/or non-monetary deprivation.

Graph. 1.1. The composite fuzzy set operations: a graphical representation of intersections



The corresponding combined measures are obtained using the Composite set operations. The Manifest deprivation propensity of individual *i* is the intersection (the smaller) of the two (similar) measures FM_i and FS_i:

$$M_i = \min(FM_i, FS_i) \tag{1.15}$$

Similarly, the Latent deprivation propensity of individual *i* is the complement of the intersection indicating the absence of both types of deprivation, i.e. the union (the larger) of the two (similar) measures FM_i and FS_i:

$$L_i = 1 - \min(\overline{FM}_i, \overline{FS}_i) = \max(FM_i, FS_i) \tag{1.16}$$

From empirical experience (Betti and Verma 2002; Betti *et al.* 2005), it appears that the degree of overlap between income poverty and non-monetary deprivation at the level of individual persons tend to be higher in poorer areas and lower in richer areas. A useful indicator in this context is the Manifest deprivation index defined as a percentage of Latent deprivation index and included between 0 and 1. When there is no overlap (i.e., when the subpopulation subject to income poverty is entirely different from the subpopulation subject to non-monetary deprivation), Manifest deprivation rate and hence the above mentioned ratio equals 0. When there is complete overlap, i.e., when each individual is subject to exactly the same degree of income poverty and of non-monetary deprivation, the Manifest and latent deprivation rates are the same and hence the above mentioned ratio equals 1.

1.4 The IFR approach

Betti *et al.* (2006) proposed a new approach to poverty and deprivation analysis, called *Integrated Fuzzy and Relative* (IFR) approach, that combines the TFR approach of Cheli e Lemmi (1995) and the approach of Betti and Verma (1999), seen in the previous section. In this approach both the share of individuals less poor than the person concerned (as in Cheli and Lemmi, 1995) and the share of the total equivalised income received by all individuals less poor than the person concerned (as in Betti and Verma 1999) are take into account. Specifically, the measure is defined as:

$$\mu_i = FM_i = (1 - F_i)^{\alpha - 1}(1 - L(F_i)) = \left(\frac{\sum_{\gamma} w_\gamma \mid y_\gamma > y_i}{\sum_{\gamma} w_\gamma \mid y_\gamma > y_1} \right)^{\alpha - 1} \left(\frac{\sum_{\gamma} w_\gamma y_\gamma \mid y_\gamma > y_i}{\sum_{\gamma} w_\gamma y_\gamma \mid y_\gamma > y_1} \right) \tag{1.17}$$

where, as in Section 1.3, y_γ is the equivalised income, F_i is the income distribution function, w_γ is the sample weight of individual of rank γ ($\gamma = 1,...,n$) in the ascending

income distribution, L_i represent the value of the Lorenz curve of income for individual *i*.

The parameter α, as in the previous approaches, is chosen so that the mean of the *m.f.* is equal to the head count ratio *H*:

$$E(FM) = \frac{\alpha + G_\alpha}{\alpha(\alpha+1)} = H \tag{1.18}$$

The Fuzzy Monetary measure as defined above is expressible in terms of the generalised Gini measure. This family of measures is a generalisation of the standard Gini coefficient with $\alpha = 1$ and it is defined (in the continuous case) as:

$$G_\alpha = \alpha(1-\alpha)\int_0^1 \{(1-F)^{\alpha-1}(F - L(F))\}dF \tag{1.19}$$

The measure in (1.19) weights the distance $(F - F(L))$ between the line of perfect equality and the Lorenz curve by a function of the individual's position in the income distribution, giving more weight to its poorer end.

Interesting applications of the IFR method have been recently reported in Betti *et al.* (2007) and Betti and Verma (2008).

In the same way as the FM indicator, a *Fuzzy Supplementary* (FS_{hi}) index for dimension *h* can be defined in two alternative manners:

iii. The proportion of individuals who are less deprived than *i*:

$$\mu_i = FS_{hi} = (1 - F_{(S),hi})^\alpha \tag{1.20}$$

where $F_{(S),hi}$ is the distribution function of *S* evaluated for individual *i* dimension *h*.

iv. The share of the total non-deprivation *S* assigned to all individuals less deprived than *i*:

$$\mu_i = FS_{hi} = (1 - L_{(S),hi})^\alpha \tag{1.21}$$

where $L_{(S),hi}$ is the value of the Lorenz curve of *S* for individual *i* in dimension *h*. The parameter α is determined so as to make the overall non-monetary deprivation rate numerically identical to the monetary poverty rate *H*.

2. Proposal for new multidimensional and fuzzy

2.1 Fuzzy Monetary Indicator

In order to calculate the Fuzzy Monetary Indicator (FM) we consider the distribution of household equivalised disposal income (variable HX090 in EU-SILC) assigned to each individual. The distribution of the equivalised disposal income is trimmed taking as low bound 15% of the median of the same distribution. This distribution is referred as *y*.

2.2 Fuzzy Supplementary Indicator

To quantify and put together diverse indicators several steps are necessary.

1. Identification of items;
2. Transformation of the items into the [0, 1] interval;
3. Exploratory and confirmatory factor analysis;
4. Calculation of weights within each dimension (each group);
5. Calculation of scores for each dimension;
6. Calculation of an overall score and the parameter α;
7. Construction of the fuzzy deprivation measure in each dimension (and overall).

2.2.1. Calculation of the deprivation score for each dimension

Aggregation over a group of items in a particular dimension *h* (*h* = 1, 2, …, *m*) is given by a weighted mean taken over *j* items: $s_{hi} = \sum w_{hj} \cdot s_{hj,i} / w_{hj}$ where w_{hj} is the weight of the *j*-th deprivation variable in the *h*-th dimension.

2.2.2. Calculation of an overall score and the parameter α

An overall score for the *i*-th individual is calculated as the unweighted mean:

$$s_i = \frac{\sum_{h=1}^{m} s_{hi}}{m} \tag{2.1}$$

Then, we calculate the FS indicator for the *i*-th individual over all dimensions as:

$$FS_i = \left(1 - F_{(S),i}\right)^{\alpha - 1}\left(1 - L_{(S),i}\right) \tag{2.2}$$

As for FM indicator, the parameter α is determined so as to make the overall non-monetary deprivation rate numerically identical to the head count ratio computed for the official poverty line (60% of the median).

The parameter α estimated is used to calculate the FS indicator for each dimension of deprivation separately.

2.2.3. Construction of the fuzzy deprivation measure in each dimension

The FS indicator for the *h*-th deprivation dimension for the *i*-th individual is defined as combination of the $(1-F_{(S),hi})$ indicator and the $(1-L_{(S),hi})$ indicator.

$$\mu_i = FS_{hi} = \left(1-F_{(S),hi}\right)^{\alpha-1}\left(1-L_{(S),hi}\right) = \left[\frac{\sum_{\gamma=i+1}^{n} w_{h\gamma} \mid s_{h\gamma} > s_{hi}}{\sum_{\gamma=2}^{n} w_{h\gamma} \mid s_{h\gamma} > s_{h1}}\right]^{\alpha-1}\left[\frac{\sum_{\gamma=i+1}^{n} w_{h\gamma}s_{h\gamma} \mid s_{h\gamma} > s_{hi}}{\sum_{\gamma=2}^{n} w_{h\gamma}s_{h\gamma} \mid s_{h\gamma} > s_{h1}}\right], \qquad (2.3)$$

$$h = 1,2,...,m; i = 1,2,...,n; \mu_{hn} = 0$$

The $(1-F_{(S),hi})$ indicator for the *i*-th individual is the proportion of individuals who are less deprived, in the *h*-th dimension, than the individual concerned. $F_{(S),hi}$ is the value of the score distribution function evaluated for individual *i* in dimension *h* and $w_{h\gamma}$ is the sample weight of the *i*-th individual of rank γ in the ascending score distribution in the *h*-th dimension.

The $(1-L_{(S),hi})$ indicator is the share of the total lack of deprivation score assigned to all individuals less deprived than the person concerned. $L_{(S),hi}$ is the value of the Lorenz curve of score in the *h*-th dimension for the *i*-th individual. The parameter α is calculated only once as shown in Section 2.2.2.

3. EU-SILC data set and identification of items

In the present work we use data from the European Survey on Income and Living Conditions (EU-SILC), distributed by Eurostat. The EU-SILC survey was designed to collect detailed information on the income of each household member, and on various aspects of the material and demographic situation of the household. A representative random sample of households throughout the country is approached to provide the required information. Data are available at cross-sectional level for years 2004, 2005, 2006 and 2007. In round 2004 only EU 15 countries are present; in rounds 2005 and

2006, 26 countries are present and in round 2007 27 countries. Below in Table 3.1 we report the number of households interviewed for each country.

3.1 Imputation of data

Missing data problems can arise from diverse sources in a number of forms. We focused on the problem of imputation for item non-response but similar problems can arise when the information is available on some but not all the members of a household.

Imputing missing data aims to minimise the mean squared error of survey estimates, in particular the non-response bias component that arises when the pattern of missing data is not random and, more practically, to reach consistency between the results from different analyses and the convenience of not having to deal with the missing data problem at the analysis stage.

Missing values of variables using in this analysis are been imputed trough IVEware (Imputation and Variance Estimation Software) and in particular IMPUTE module. This is a multivariate imputation procedure that can handle relatively complex data structures (hundreds of variables, some continuous, others counts, many dichotomous or polytomous, and semi-continuous or limited dependent variables) when the data are missing at random.

IMPUTE produces imputed values for each individual in the data set conditional on all the values observed for that individual. The imputations are obtained by fitting a sequence of regression models; they depend on the type of variable being imputed, and drawing values from the corresponding predictive distributions specified by the regression model with a flat or non-informative prior distribution for the parameters in the regression model. Covariates include all other variables observed or imputed for that individual. The sequence of imputing missing values can be continued in a cyclical manner, each time overwriting previously drawn values, building interdependence among imputed values and exploiting the correlational structure among covariates. To generate multiple imputations, the same procedure can be applied with different random starting seeds or taking every p-th imputed set of values in the cycles mentioned above.

Five types of variables are assumed: (1) continuous; (2) binary; (3) categorical (polytomous with more than two categories); (4) counts; and (5) mixed (a continuous variable with a non-zero probability mass at zero). The types of regression models used

are linear, logistic, Poisson, generalized logit or mixed logistic/linear, depending on the type of variable being imputed. IMPUTE take also into account two common features of survey data that add to the complexity of the modelling process: the restriction of imputations to subpopulations, and the bounding of imputed values. For details see Raghunathan *et al*. (2001).

3.2 Identification of items

Firstly, from the large set of EU-SILC variables, a selection has been made of indicators which are substantively meaningful and useful for the construction of Fuzzy Supplementary Indicators. For our purpose, we have identified a set of items which could serve as indicators of concept of life-style deprivation. All these items are considered at household level, even if some of them are taken from the individual dataset and then aggregated to household level.

Table 3.1. EU-SILC household sample sizes. Waves 2004-2007

Country	**2004**	**2005**	**2006**	**2007**
AT	4,521	5,148	6,028	6,806
BE	5,275	5,137	5,860	6,348
CY		3,746	3,621	3,505
CZ		4,351	7,483	9,675
DE		13,106	13,799	14,153
DK	6,866	5,957	5,711	5,783
EE	3,993	4,169	5,631	5,146
ES	15,355	12,996	12,205	12,329
FI	11,200	11,229	10,868	10,624
FR	10,273	9,754	10,036	10,498
GR	6,252	5,568	5,700	5,643
HU		6,927	7,722	8,737
IE	5,477	6,085	5,836	5,608
IS	2,907	2,928	2,845	2,872
IT	24,270	22,032	21,499	20,982
LT		4,441	4,660	4,975
LU	3,571	3,622	3,836	3,885
LV		3,843	4,315	4,471
MT				3,477
NL		9,356	8,986	10,219
NO	6,046	5,991	5,768	6,013
PL		16,263	14,914	14,286
PT	4,989	4,615	4,367	4,310
SE	5,748	6,133	6,803	7,183
SI		8,287	9,478	8,707
SK		5,147	5,105	4,941
UK		10,826	9,902	9,275
TOT	**116,743**	**197,657**	**202,978**	**210,451**

The first set of items regards the lack of possession of a widely-desired item. These are:

- A telephone including mobile phone;
- A colour TV;
- A computer;
- A washing machine;
- A car.

In all these cases we consider a household to be deprived only if the lack of the item is enforced, in the sense that the household would like to have the item but cannot afford it. A second set of items relates to the lack of ability to afford items that are considered as basic:

- Keeping home adequately warm;
- Paying for one week annual holiday away from home;
- Eating a meal with meat, chicken, fish (or vegetarian equivalent) every second day;
- Being able to meet unexpected financial expenses.

A third set relates to absence of housing facilities, considered so basic that one can presume all household to wish to have them:

- A bath or shower in dwelling;
- An indoor flushing toilet for sole use of the household.

The fourth set of items relates to problems with accommodation and the environment, with the implicit assumption that the households wish to avoid such difficulties:

- Leaking roof, damp walls/floors/foundation, or rot in window frames or floor;
- Too dark, not enough light in dwelling;
- Noise from neighbours or from the street;
- Pollution, grime or other environmental problems;
- Crime violence or vandalism in the area.

The fifth set relates to arrears in paying bills that the household has experienced in the last 12 months;

- Arrears on mortgage or rent payments;
- Arrears on utility bills;
- Arrears on hire purchase instalments or other loan payments.

The sixth set is just one item related to the capacity of the household to make ends meet.

The seventh set relates to the health condition of the household. These items are from individual variables that have been aggregated at household level. We consider this dimension because we think that, in dealing with life-style deprivation, a lack of good health is also important. The items considered are:

- General health;
- Suffer from any chronic (long-standing) illness or condition;
- Limitation in activities because of health problems;
- Unmet need for medical examination or treatment;
- Unmet need for dental examination or treatment.
- This dimension is not comparable for register countries, for which the unit of analysis is just the selected respondent.

The eighth set relates to the education. For this set we have constructed two composite indicators:

- Households with early school leavers not in education or training;

  Households with at least one person aged 18-24 with only lower secondary education or less (PE040: ISCED level currently attended: value 2 or less), and who at the same time is not in education or training leading to a qualification at least to upper secondary level (PE010: current education activity: value 2)
- Households with persons with low educational attainment.

  Households with at least one person aged 25-64 who has only lower secondary education or less (PE040).

The least dimension concerns the labour market. Also for this set we have constructed two composite indicators:

- Jobless households;

This indicator identifies the worklessness of the household, using variable PL030. For details about the construction see next section.

- Intensity or duration of unemployment at household level.

This indicator is constructed using variables PL070, PL072, PL080, PL085, PL087, PL090. For details about the construction see next section.

The variables used are listed below:

HH040: Leaking Roof, Damp Walls/Floors/Foundation, Or Rot In Window Frames Or Floor
HH050: Ability to keep home adequately warm

HH080: Bath or shower in dwelling
HH090: Indoor flushing toilet for sole use of household
HS010: Arrears on mortgage or rent payments
HS020: Arrears on utility bills
HS030: Arrears on hire purchase instalments or other loan payments
HS040: Capacity to afford paying for one week annual holiday away from home
HS050: Capacity to afford a meal with meat, chicken, fish (or vegetarian equivalent) every second day
HS060: Capacity to face unexpected financial expenses
HS070: Do you have a telephone (including mobile phone)?
HS080: Do you have a colour TV?
HS090: Do you have a computer?
HS100: Do you have a washing machine?
HS110: Do you have a car?
HS120: Ability to make ends meet
HS160: Problems with the dwelling: too dark, not enough light
HS170: Noise from neighbors or from the street
HS180: Pollution, grime or other environmental problems
HS190: Crime violence or vandalism in the area
PE010: CUrrent EDUCATION ACTIVITY
PE040: HIGHEST ISCED LEVEL ATTAINED
PH010: General health
PH020: Suffer from any a chronic (long-standing) illness or condition
PH030: Limitation in activities because of health problems
PH040: Unmet need for medical examination or treatment
PH060: Unmet need for dental examination or treatment
PL030: SELF-DEFINED CURRENT ECONOMIC STATUS
PL070: Number of months spent at full-time work
PL072: Number of months spent at part-time work
PL080: Number of months spent in unemployment
PL085: Number of months spent in retirement
PL087: Number of months spent studying
PL090: Number of months spent in inactivity

4. Transformation of the items into the [0, 1] interval

When the item is constituted by a fixed number of categories, then it is transformed using the following procedure. For each item we determine a deprivation score as follows:

$$d_{j,i} = \frac{1 - F(c_{j,i})}{1 - F(1)}; \; j = 1, 2, ..., k; \; i = 1, 2, ..., n \qquad (4.1)$$

where $c_{j,i}$ is the value of the category of the *j*-th item for the *i*-th individual and $F(c_{j,i})$ is the value of the *j*-th item cumulation function for the *i*-th individual.

We transform the deprivation score to a positive score as follows:

$$s_{j,i} = 1 - \frac{1 - F(c_{j,i})}{1 - F(1)} = \frac{F(c_{j,i}) - F(1)}{1 - F(1)}; \; j = 1, 2, ..., k; i = 1, 2, ..., n \qquad (4.2)$$

In the special, but the common case, where the variable is a dichotomy, the deprivation index *d* is 1 for deprivation and 0 otherwise, while the positive score *s* is 0 for deprivation and 1 otherwise. For a polychotomous item we assign to each household instead of the real value of the category, a value corresponding to the percentage of households that are "better off" than that household.

In the few cases in which the indicator is a composite one (a set of dichotomies indicating the presence or absence of an experience by household members), the score *s* represents the proportion of people in the household that experienced it.

The indicator concerning the worklessness of the household is constructed as follows. First we exclude households consisting only of persons who are aged 18-24 in full-time education or are older than a country-specific retirement age. In order to choose an appropriate retirement age we have proceeded as follows. Among people that have ever worked, we consider the distribution of the ones that are retired (PL030=5) by age and gender. Looking at the ratio of people that at a particular age are retired among all the people in that age, we look for the age where a large jump in this proportion is found to occur. Once this point has been found, we confirmed it by examining its relationship to the legal age of retirement for a specific country.

Among the remaining households we classify the people as employed or not employed using variable PL030. We thus identify the degree of worklessness of an household, by constructing a ratio where in the numerator there are all the people in the household for which variable PL030 takes value 1, 2 or 7, and the denominator is the sum of the people of the household for which PL030 takes value 1, 2, 3, 6, 7, 9, and values 5 and 8 only if the age of the person is less then the retirement age chosen above. So at household level we construct an index reflecting the degree of which eligible household members are engaged in work: a zero indicates a workless household with some of its members in working age; a one indicates that all its working age members are working.

To construct the indicator concerning the duration of unemployment, we calculate at household level the ratio:

$$1 - \frac{\sum_{ind=1}^{HH-size} PL080_{ind}}{\sum_{ind=1}^{HH-size} (\text{PL070} + \text{PL072} + \text{PL080} + \text{PL085} + \text{PL087} + \text{PL090})_{ind}} \qquad (4.3)$$

The variable for general health, PH010, is aggregated as follows. To the categories 1-2-3 is assigned value 1 and to categories 4-5 value 0. Then this variable is aggregated at household level so that a household is considered deprived for that indicator if at least one person in the household is deprived for the item. So the score *s* assumes value 1 if no one in the household is deprived concerning that item, and it assumes value 0 is at least one person is deprived.

The same kind of household aggregation is done for all the personal variables concerning the health and the educational status.

5. Factor analysis

In order to investigate on life-style deprivation we have followed the procedure from the Economic and Social Research Ireland (ESRI), as described in Whelan *et al.* (2001).

In proceeding to construct a summary index of deprivation employing different items, we begin by identifying and investigating the dimension of deprivation. By 'dimension' we mean a distinct group of individual items of deprivation. Exploratory and confirmatory factor analyses allow us to achieve this objective. The procedure consists in an exploratory factor analysis to give a preliminary framework of the dimensions. We then proceed to rearrange some factors in the dimensions found in order to create more meaningful groups. Finally, we do a confirmatory factor analysis to test the goodness of the model hypothesised.

The exploratory factor analysis identifies 9 dimensions as reported in Table 5.1.

Then we decided to rearrange the dimensions in order to achieve substantially more meaningful groupings, as reported in Table 5.2.

Table 5.1. Results of exploratory factor analysis.

INDICATORS	DIMENSIONS
- Meals with meat, fish or chicken - Household adequately warm - Holiday away from home - Inability to cope with unexpected expenses - Ability to make ends meet	**1**
- General health - Chronic illness - Mobility restriction	**2**
- Pollution - Crime, Violence, vandalism - Noise	**3**
- Bath or Shower - Indoor flushing toilet	**4**
- Car - PC - Telephone - Washing Machine - TV	**5**
- Worklessness - Duration of unemployment - Early school livers - Low education	**6**
- Arrears on mortgage or rent payments - Arrears on utility bills - Arrears on hire purchase instalments	**7**
- Unmet need for medical exam. - Unmet need for dental exam.	**8**
- Leaking roof and damp - Rooms to dark	**9**

Table 5.2. Dimensions after rearrangement and confirmatory factor analysis.

INDICATORS	REARRANGEMENT of the dimensions	Name
1 Meals with meat, fish or chicken **2** Household adequately warm **3** Holiday away from home **4** Ability to make ends meet	**1**	Basic lifestyle
5 Car **6** PC **7** Telephone **8** Washing Machine **9** TV	**2**	Consumer durables
10 Bath or Shower **11** Indoor flushing toilet **12** Leaking roof and damp **13** Rooms to dark	**3**	Housing amenities
14 Inability to cope with unexpected expenses **15** Arrears on mortgage or rent payments **16** Arrears on utility bills **17** Arrears on hire purchase instalments	**4**	Financial situation
18 Crime, Violence, vandalism **19** Pollution **20** Noise	**5**	Environment
21 Early school livers **22** Low education **23** Worklessness **24** Duration of unemployment	**6**	Work & Education
25 General health **26** Chronic illness **27** Mobility restriction **28** Unmet need for medical exam. **29** Unmet need for dental exam.	**7**	Health related

In summary the seven final dimensions are:

1 Basic life-style – these concern the lack of ability to afford most basic requirements:

- Keeping the home (household's principal accommodation) adequately warm.
- Paying for a week's annual holiday away from home.
- Eating meat chicken or fish every second day, if the household wanted to.
- Ability to make ends meet

2 Consumer durables - these concern enforced lack of widely desired possessions ("enforced" means that the lack of possession is because of lack of resources)

- A car or van.
- A colour TV.
- A pc
- A washing machine.
- A telephone.

3 Housing amenities – these concern the absence of basic housing facilities (so basic that one can presume all households would wish to have them):

- A bath or shower.
- An indoor flushing toilet.
- Leaking roof and lamp
- Rooms to dark

4 Financial situation – these concern the lack of ability to pay in time due to financial difficulties:

- Inability to cope with unexpected expenses.
- Arrears on mortgage or rent payments.
- Arrears on utility bills.
- Arrears on hire purchase instalments.

5 Environmental problems – these concern problems with the neighbourhood and the environment:

- Pollution.
- Crime, violence, vandalism.
- Noise.

6 Work and education – these concern the absence of education and job

- Households with early school livers not in education or training.
- Households with persons with low educational attainment.
- Jobless households.
- Intensity or duration of unemployment at household level.

7 Health related – these concern problems with personal health:

- General health.
- Chronic illness.
- Mobility restriction.
- Unmet need for medical examination or treatment.
- Unmet need for dental examination or treatment.

Subsequently we applied the confirmatory factor analysis to the dimensions rearranged as above.

The results of the analysis are very good; in fact all the indicators of goodness of the model are significant. Below, we report measures of absolute, relative and parsimonious fit as follows:

- The Goodness of Fit Index (GFI) is 0.94. It is based on the ratio of the sum of

squared discrepancies to the observed variances; it ranges from 0 to 1 with values above 0.9 indicating a good fit.

- The Adjusted Goodness of Fit Index (AGFI) is 0.93. It is the GFI adjusted for degrees of freedom of the model, that is the number of the fixed parameters. It can be interpreted in the same manner.
- The Parsimonious GFI is 0.86. It adjusts GFI for the number of estimated parameters in the model and the number of data points.
- The Root Mean Square Residual (RMR) is 0.06. The fit is considered really good if RMR is equal or below 0.06.
- The Root Mean Squared Error of Approximation (RMSEA) is 0.0475. It is based on the analysis of residuals, with small values indicating a good fit. Values below 0.1, 0.05 and 0.01 indicate a good, very good and outstanding fit respectively.

6. Calculation of weights within each dimension

The weights to be given to items are determined within each dimension separately and the set of weights are taken to be item-specific, i.e. for a given item they are common to all individuals in the population. Such weights comprise two factors: the dispersion of deprivation indicator and its correlation with other deprivation indicators in the given dimension:

$$w_{hj} = w_{hj}^{a} \cdot w_{hj}^{b}, h = 1, 2, ..., m;\ j = 1, 2, ..., k_h \qquad (6.1)$$

where *h* is a particular dimension and *j* a particular deprivation indicator.

In a previous work, the first factor w_{hj}^{a} has been taken as proportional to the coefficient of variation of deprivation score for the variable concerned, $w_{hj}^{a} \propto cv_{hj}$ (Betti and Verma, 1999).

Here the indicators were in terms of deprivation indexes *d*, defined above. This means that when an item of deprivation affects only a small proportion, the weight given to it varies inversely to the square-root of the proportion. Thus deprivation affecting a small proportion of the population is treated as more intense at the individual person's level but, of course, its contribution to the average level of deprivation in the population as a whole is correspondingly smaller.

Now our analysis is carried on using the deprivation scores *s*, so the previous formula should be modified as follows:

$$w_{hj}^{a} \propto \frac{std_{hj}}{1-mean_{hj}} \tag{6.2}$$

The second factor, as a measure of the correlation, can be computed in the following form:

$$w_{hj}^{b} \propto \left(\frac{1}{1+\sum_{j=1}^{k_h} r_{e_{hj,hj'}} \mid r_{e_{hj,hj'}} < r_{e_{hj}}^{*}} \right) * \left(\frac{1}{\sum_{j=1}^{k_h} r_{e_{hj,hj'}} \mid r_{e_{hj,hj'}} \geq r_{e_{hj}}^{*}} \right) \tag{6.3}$$

where $r_{e_{hj,hj'}}$ is the correlation coefficient between deprivation indicators corresponding to items *j* and j' in the *h*-dimension and $r_{e_{hj}}^{*}$ is the critical value of the correlation coefficient.

Below in Table 6.1 we report the results from our data for w_{hj}, where, as noted earlier, h refers to the dimension and j to a particular item of deprivation in it.

Table 6.1. The weighting system.

Country	W11	W12	W13	W14	W21	W22	W23	W24	W25	W31	W32	W33	W34	W41	W42	W43	W44	W51	W52	W53	W61	W62	W63	W64	W71	W72	W73	W74	W75
AT	1.56	3.78	0.84	0.40	2.22	2.41	11.60	8.27	10.11	5.13	4.42	2.48	3.01	1.06	3.92	3.85	5.23	2.11	1.91	1.25	5.08	1.56	0.60	2.20	1.21	0.58	0.52	3.43	2.96
BE	2.67	1.31	0.80	0.38	1.60	1.69	10.85	2.92	6.82	5.07	6.44	1.93	2.55	1.11	3.37	2.50	6.40	1.36	1.51	1.15	5.28	1.05	0.47	1.49	1.20	0.57	0.60	5.67	3.18
CY	1.75	0.64	0.43	0.32	4.32	3.02	12.64	5.25	11.34	3.12	3.14	1.09	3.04	0.69	2.86	1.79	1.46	0.98	1.81	0.74	4.19	0.89	0.45	2.07	0.93	0.43	0.56	1.47	1.15
CZ	1.28	2.06	0.66	0.37	1.49	1.75	5.49	7.06	8.67	5.15	4.24	1.68	3.34	0.86	2.68	2.55	5.03	1.25	1.54	1.26	6.18	1.96	0.54	1.86	0.84	0.44	0.46	1.94	2.33
DE	1.35	2.15	0.82	0.39	1.96	2.64	9.66	7.69	7.06	8.43	6.34	2.19	3.89	0.87	3.68	2.75	4.59	0.99	1.62	0.88	7.32	2.55	1.11	1.94	1.32	0.52	0.58	1.78	1.61
DK	3.62	1.94	1.56	0.51	1.72	3.73		4.59	7.27	9.01		2.60	4.06	1.29	3.36	3.45	2.65	2.21	1.82	1.28	5.55	1.33	0.62	4.06					
EE	2.02	2.98	0.45	0.47	0.91	1.16	3.36	2.05	5.51	0.90	1.04	1.04	2.09	1.27	6.81	2.76	7.94	1.07	1.38	1.14	3.80	1.89	0.57	2.41	0.74	0.34	0.37	1.05	1.08
ES	4.27	2.00	0.71	0.41	3.41	2.52	9.73	13.07	16.68	9.68	11.06	1.67	2.32	1.05	3.65	3.12	4.58	1.50	1.50	1.08	2.94	0.59	0.41	1.59	0.91	0.54	0.55	4.28	1.87
FI	3.31	6.45	1.17	0.47	1.29	1.90	14.13	3.27	4.94	4.92	5.92	3.73	3.92	0.88	3.16	3.25	4.06	1.61	1.71	1.48	6.12	1.49	0.56	2.00					
FR	1.97	2.46	0.73	0.39	3.21	2.30	7.64	5.56	14.94	5.64	5.47	1.90	2.44	0.84	2.09	1.91	3.61	1.41	1.51	1.26	5.07	1.07	0.57	1.92	1.19	0.49	0.63	2.74	1.87
GR	1.69	1.13	0.46	0.31	1.71	1.71	6.06	3.23	8.20	4.75	2.96	1.22	2.40	0.86	2.22	1.24	2.10	1.12	1.73	1.07	5.68	0.72	0.44	1.74	0.94	0.50	0.56	1.41	1.52
HU	0.82	1.55	0.37	0.40	0.97	1.21	2.98	2.79	6.59	2.07	1.83	1.10	1.68	0.54	3.49	1.24	2.85	1.53	1.70	1.41	4.66	1.21	0.51	1.97	0.52	0.33	0.40	1.14	1.28
IE	4.09	2.98	1.05	0.42	2.06	2.16	8.18	6.13	12.24	5.85	6.24	1.67	2.28	0.78	2.34	1.88	3.55	2.04	1.57	1.57	4.44	0.69	0.41	2.04	2.51	0.56	0.67	2.83	2.54
IS	2.97	2.45	1.38	0.46	5.87	6.55		49.91	16.61	30.94	11.13	3.03	7.09	0.99	1.90	2.06	1.53	2.01	4.09	1.86	3.29	1.23	0.56	4.86					
IT	1.95	1.43	0.59	0.38	2.72	2.06	4.81	6.46	8.75	9.60	13.82	1.35	2.31	0.87	3.13	1.64	3.77	1.03	1.38	0.95	4.11	0.69	0.48	2.04	0.92	0.58	0.50	1.61	1.36
LT	1.01	0.92	0.37	0.39	1.19	1.50	2.43	1.62	3.98	0.84	0.84	1.03	1.81	0.85	8.01	2.18	8.28	1.29	2.31	1.14	6.30	1.97	0.55	2.24	0.63	0.39	0.45	1.10	1.32
LU	3.86	6.67	1.49	0.48	3.54	3.16	16.45	9.25	21.62	7.11	6.45	1.98	3.25	1.33	3.60	3.37	8.78	1.31	1.93	1.12	4.24	0.92	0.52	2.76	1.41	0.65	0.68	2.85	2.82
LV	0.64	0.83	0.32	0.32	0.75	0.92	2.38	1.52	4.23	0.78	0.87	0.92	1.67	0.60	3.95	1.96	6.97	0.78	0.95	1.17	3.53	1.45	0.54	2.13	0.57	0.34	0.36	0.62	0.74
NL	4.31	4.13	1.18	0.54	2.41	4.62		17.66	41.24	12.64	30.72	1.89	3.51	1.09	2.95	3.42	4.94	1.83	1.59	0.98	5.32	1.13	0.55	2.54					
NO	2.78	5.69	1.73	0.46	2.13	3.83	15.94	10.36	8.47	11.73	8.88	2.76	4.06	1.27	1.89	1.82	2.62	2.19	3.27	1.61	3.68	1.53	0.63	4.71					
PL	0.77	0.82	0.34	0.32	1.21	1.27	3.15	4.95	5.34	1.38	1.53	0.77	1.82	0.65	7.17	1.43	3.36	1.42	2.00	1.11	3.70	1.23	0.44	1.65	0.67	0.40	0.50	1.09	1.31
PT	3.34	0.57	0.38	0.37	1.57	1.59	2.33	2.65	6.25	2.20	2.34	1.27	1.58	1.42	3.58	2.54	5.01	1.15	1.82	1.00	2.61	0.42	0.42	1.59	0.55	0.37	0.39	1.13	1.78
SE	2.75	4.21	1.32	0.44	3.19	4.60		14.87	8.37	10.03		3.45	3.28	1.24	3.46	2.85	3.28	2.54	1.99	1.74	7.88	1.92	0.63	3.11					
SI	1.45	2.70	0.72	0.40	2.29	2.02	5.82	8.37	5.87	4.31	4.46	1.66	2.40	0.77	4.75	1.69	2.94	1.22	2.02	1.24	6.84	1.15	0.50	2.13					
SK	0.71	2.87	0.44	0.41	0.98	1.21	3.71	5.13	5.60	3.71	2.54	2.39	2.97	0.76	3.03	2.20	4.56	1.23	2.20	1.18	7.12	2.16	0.54	1.88	0.60	0.45	0.43	1.97	2.02
UK	2.63	2.55	0.99	0.43	2.80	3.02	17.27	10.59	27.69	23.09	10.72	2.14	2.46	1.00	2.74	2.49	4.15	1.82	1.18	1.32	7.59	1.65	0.59	4.24	1.62	0.46	0.64	2.71	2.89

7. Empirical analysis

Fuzzy measures of monetary poverty and non-monetary deprivation have been constructed, step by step as described in the previous sections, based on EU-SILC survey data. A cross-sectional analysis have been conducted from 2004 to 2007 waves. Figure 7.1 shows cartograms of fuzzy monetary indicators (equal to HCR and to the overall non-monetary index) in European Countries. Differences between these years are not so very significant.

Figure 7.1. Cartograms of fuzzy monetary indicators in European Countries (2004-2007)

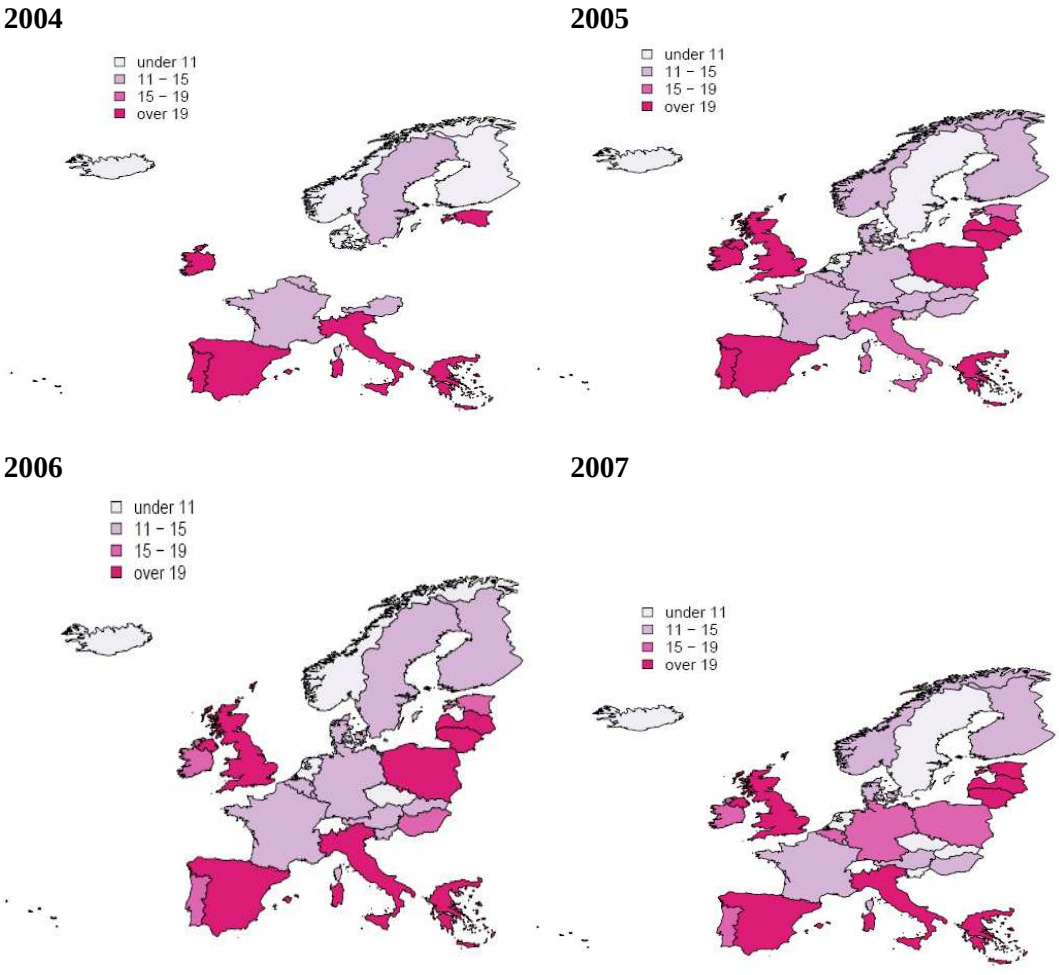


Table 7.1 shows indicators of deprivation in various dimensions estimated with the methodology described above, using EU-SILC 2007 data.

The objective of illustrating those results is both substantive and methodological. It shows the relative situation of EU countries in terms of levels of overall deprivation

(monetary and non-monetary), and also in terms of different dimensions of deprivation. At the same time, the table illustrates the type of numerical values obtained with the above procedure, thus further clarifying details of the methodology.

Denmark, Finland, Island, Norway, Sweden, Slovenia and Netherland are register Countries, then as explained in Section 2.4.4, they miss health dimension (FS7).

The first column, FS0, is the overall deprivation rate. It is in fact the conventional poverty rate (HCR) for each country. The values of the FM (fuzzy monetary) and FS (fuzzy supplementary) deprivation indices are simply scaled for each country to numerically equal the conventional HCR.

Those overall poverty or deprivation rates show large differences among EU countries, from the low value of 9.5% in CZ to the high of 21.2% in LV. In six countries the rate is below 11% (CZ, IS, NL, SK, SE, SI), it exceeds 19% in seven (LV, GR, IT, ES, EE, LT, UK). The average over countries is close to 15%.

We note that there is fairly strong correlation between the ranking of countries according to the overall and dimension-specific indices of deprivation. However, quite large differences in the rankings according to different dimensions are also present.

Numerically, deprivation rates for individual dimensions are not scaled in the methodology described above to equal – individually or even in the average over dimensions – the overall poverty or deprivation rate FS0. In fact, over countries, in these data the average of rates for individual dimensions (at 11%) is lower than the average of overall rates (15%).

In certain dimensions, the average over countries is 12-14%, which is quite close to that for the overall index (15%). This group includes:

FS1 – basic life-style

FS5 – environment

FS6 – work and education

FS7 – health related

For the remaining dimensions, the average values obtained are much lower (7-9%). These dimensions are:

FS2 – consumer durables

FS3 – housing amenities

FS4 – financial situation

Table 7.1. Fuzzy measures at Country level, SILC 2007 wave.

	Rate of deprivaton by dimension of deprivation								mean	'Normalised rates'								mean
Country	**FS0**	**FS1**	**FS2**	**FS3**	**FS4**	**FS5**	**FS6**	**FS7**	**FS1-FS7**	**FS0**	**FS1**	**FS2**	**FS3**	**FS4**	**FS5**	**FS6**	**FS7**	**FS1-FS7**
CZ	0.095	0.092	0.061	0.055	0.045	0.106	0.087	0.085	0.076	1.00	1.17	1.34	0.97	0.86	1.26	1.11	0.95	1.093
IS	0.100	0.087	0.021	0.041	0.084	0.071	0.083		0.065	1.00	1.05	0.45	0.70	1.53	0.81	1.02		*0.927*
NL	0.102	0.080	0.040	0.051	0.051	0.097	0.087		0.068	1.00	0.95	0.82	0.84	0.92	1.08	1.04		*0.943*
SK	0.105	0.087	0.063	0.059	0.055	0.103	0.094	0.095	0.079	1.00	1.01	1.26	0.96	0.96	1.11	1.09	0.96	1.050
SE	0.107	0.085	0.040	0.058	0.065	0.085	0.089		0.070	1.00	0.96	0.79	0.92	1.10	0.90	1.01		*0.946*
SI	0.109	0.094	0.052	0.066	0.075	0.100	0.093		0.080	1.00	1.04	1.00	1.02	1.26	1.04	1.04		1.068
DK	0.117	0.099	0.057	0.064	0.062	0.100	0.093		0.079	1.00	1.03	1.02	0.93	0.97	0.97	0.97		*0.980*
AT	0.120	0.098	0.058	0.070	0.047	0.102	0.105	0.088	0.081	1.00	0.98	1.01	0.98	0.72	0.96	1.06	0.78	*0.929*
NO	0.123	0.082	0.044	0.058	0.085	0.084	0.100		0.076	1.00	0.80	0.75	0.80	1.27	0.78	0.99		*0.900*
HU	0.124	0.127	0.085	0.096	0.083	0.112	0.106	0.140	0.107	1.00	1.24	1.44	1.31	1.22	1.03	1.04	1.20	1.212
FI	0.130	0.097	0.067	0.063	0.075	0.112	0.110		0.087	1.00	0.90	1.08	0.83	1.05	0.98	1.03		*0.979*
FR	0.131	0.101	0.058	0.078	0.078	0.126	0.111	0.107	0.094	1.00	0.93	0.94	1.00	1.08	1.09	1.02	0.87	*0.988*
LU	0.135	0.092	0.028	0.071	0.055	0.119	0.110	0.106	0.083	1.00	0.82	0.43	0.89	0.74	1.00	0.99	0.84	*0.816*
BE	0.151	0.131	0.071	0.087	0.081	0.141	0.127	0.102	0.105	1.00	1.04	0.98	0.97	0.98	1.05	1.02	0.72	*0.966*
DE	0.152	0.124	0.058	0.079	0.063	0.145	0.119	0.130	0.103	1.00	0.99	0.81	0.88	0.76	1.08	0.95	0.92	*0.912*
CY	0.155	0.140	0.058	0.075	0.117	0.146	0.128	0.143	0.115	1.00	1.09	0.79	0.81	1.37	1.06	1.00	0.98	1.014
PL	0.173	0.200	0.105	0.113	0.094	0.135	0.146	0.167	0.137	1.00	1.39	1.27	1.10	0.99	0.88	1.03	1.03	1.097
IE	0.175	0.128	0.083	0.095	0.086	0.133	0.143	0.124	0.113	1.00	0.89	1.00	0.92	0.90	0.86	0.99	0.76	*0.902*
PT	0.181	0.130	0.115	0.119	0.097	0.158	0.151	0.154	0.132	1.00	0.86	1.33	1.10	0.97	0.99	1.01	0.90	1.024
UK	0.191	0.143	0.060	0.103	0.105	0.162	0.146	0.137	0.122	1.00	0.91	0.66	0.91	1.00	0.96	0.93	0.76	*0.875*
LT	0.191	0.167	0.124	0.158	0.082	0.143	0.152	0.176	0.143	1.00	1.05	1.36	1.39	0.78	0.84	0.97	0.98	1.054
EE	0.194	0.126	0.114	0.149	0.090	0.183	0.155	0.181	0.143	1.00	0.79	1.23	1.30	0.85	1.07	0.97	1.00	1.030
ES	0.197	0.145	0.073	0.103	0.095	0.172	0.163	0.143	0.128	1.00	0.89	0.78	0.88	0.88	0.99	1.00	0.77	*0.884*
IT	0.198	0.164	0.064	0.100	0.117	0.192	0.155	0.169	0.137	1.00	1.00	0.68	0.85	1.08	1.10	0.95	0.91	*0.937*
GR	0.203	0.165	0.109	0.113	0.152	0.169	0.160	0.165	0.148	1.00	0.99	1.13	0.94	1.37	0.94	0.96	0.87	1.028
LV	0.212	0.219	0.136	0.171	0.081	0.224	0.169	0.246	0.178	1.00	1.25	1.35	1.37	0.70	1.20	0.97	1.24	1.154
average	0.149	0.123	0.071	0.088	0.081	0.132	0.122	0.140	0.108	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

NOTES FS0 stands for "HCR = FM = FS"
'Normalised rates' Nij: all values scaled such that:
(1) for each dimension (j), average over countries rescaled to = 1.0; and
(2) for each country (i), FSj values scaled to correspond to FS0 = 1.0.

$$N_{ij} = \left(\frac{FS_{ij}}{FS_{.j}} \right) \Big/ \left(\frac{FS_{i0}}{FS_{.0}} \right)$$

FS1 – FS7 refer to the seven dimensions of deprivation defined in Table 5.2.

We believe that the indices for individual dimensions represent a mixture of relative and absolute levels of deprivation, even if the relative aspect predominates. However, values observed for dimensions 2-4 imply that, compared to overall deprivation and to other dimensions, deprivation in these dimensions may be less severe in the absolute sense in EU countries on the average.

The second panel of Table 7.1 examines the pattern of variation across countries and dimensions more closely, bringing out the relationship in scores across different dimensions in relative terms.

The figures shown are ‘normalised’, meaning that we have rescaled them to remove the effect of variations among countries in the overall deprivation (or poverty) rates FS0,

and also to remove the effect of differing average values for the various dimensions.
The last column shows the average over the dimensions (FS1-FS7) of the 'normalised' values. This average, by definition, is 1.0 over all countries.
The overall non-monetary dimension and each of the seven non-monetary dimensions have been combined with the monetary dimension in order to obtain measures of manifest and latent deprivation which correspond respectively to intersection and union of the fuzzy sets. Table 7.2 reports values of latent and manifest deprivation for aggregated measures of overall deprivation and the combination of the monetary dimension with each of the seven non-monetary dimensions. The M0/L0 ratio is in general lower in areas with lower levels of deprivation (for example IS and NL), and higher in areas with higher levels (LV and GR). High values of this ratio imply that different types of deprivation overlap and this means that deprivation in the income and non-monetary domains is more likely to afflict the same individuals in the population. On the other hand, low values imply the absence of such overlap at the micro level. Analogously, for each dimension, the overlap between monetary and non-monetary deprivation increases for Countries with higher levels of poverty and deprivation, even if the ranking is not so sharp and there are some exceptions like CK in the second and sixth dimensions and LV in the forth dimension.

Table 7.2. Latent and Manifest deprivation at aggregated level and for each dimension of deprivation (2007)

Country	FS	L0	M0	*M0/L0*	L1	M1	*M1/L1*	L2	M2	*M2/L2*	L3	M3	*M3/L3*	L4	M4	*M4/L4*	L5	M5	*M5/L5*	L6	M6	*M6/L6*	L7	M7	*M7/L7*
CZ	0.095	0.154	0.037	0.238	0.153	0.034	*0.224*	0.126	0.029	*0.233*	0.133	0.017	*0.128*	0.124	0.016	*0.132*	0.183	0.018	*0.100*	0.145	0.037	*0.254*	0.166	0.014	*0.084*
IS	0.100	0.171	0.028	0.162	0.162	0.024	*0.151*	0.114	0.007	*0.058*	0.131	0.010	*0.075*	0.163	0.021	*0.127*	0.157	0.014	*0.090*	0.168	0.015	*0.092*			
NL	0.102	0.174	0.030	0.171	0.150	0.031	*0.209*	0.124	0.017	*0.140*	0.141	0.012	*0.085*	0.134	0.020	*0.146*	0.181	0.018	*0.099*	0.162	0.027	*0.169*			
SK	0.105	0.170	0.039	0.231	0.156	0.036	*0.232*	0.140	0.027	*0.195*	0.147	0.017	*0.118*	0.141	0.019	*0.136*	0.190	0.018	*0.094*	0.160	0.039	*0.241*	0.177	0.022	*0.127*
SE	0.107	0.180	0.034	0.188	0.160	0.032	*0.200*	0.131	0.016	*0.122*	0.153	0.013	*0.084*	0.148	0.024	*0.162*	0.176	0.017	*0.095*	0.172	0.024	*0.139*			
SI	0.109	0.178	0.039	0.216	0.167	0.035	*0.212*	0.137	0.023	*0.170*	0.153	0.022	*0.142*	0.161	0.023	*0.143*	0.191	0.018	*0.094*	0.164	0.037	*0.225*			
DK	0.117	0.192	0.041	0.213	0.180	0.036	*0.198*	0.150	0.024	*0.157*	0.165	0.016	*0.097*	0.153	0.025	*0.164*	0.191	0.025	*0.133*	0.185	0.026	*0.139*			
AT	0.120	0.196	0.044	0.227	0.175	0.043	*0.246*	0.153	0.025	*0.165*	0.169	0.021	*0.123*	0.148	0.019	*0.130*	0.200	0.023	*0.113*	0.186	0.039	*0.212*	0.186	0.023	*0.122*
NO	0.123	0.204	0.042	0.204	0.172	0.033	*0.189*	0.146	0.021	*0.140*	0.165	0.016	*0.096*	0.179	0.029	*0.162*	0.188	0.019	*0.102*	0.193	0.031	*0.160*			
HU	0.124	0.196	0.051	0.262	0.205	0.046	*0.223*	0.174	0.035	*0.202*	0.185	0.034	*0.184*	0.174	0.032	*0.183*	0.211	0.025	*0.118*	0.180	0.049	*0.272*	0.233	0.030	*0.131*
FI	0.130	0.212	0.048	0.226	0.182	0.045	*0.246*	0.162	0.034	*0.211*	0.177	0.017	*0.095*	0.173	0.032	*0.184*	0.215	0.027	*0.123*	0.202	0.038	*0.186*			
FR	0.131	0.209	0.054	0.259	0.186	0.046	*0.246*	0.165	0.024	*0.148*	0.184	0.025	*0.134*	0.178	0.031	*0.172*	0.226	0.031	*0.139*	0.194	0.048	*0.246*	0.205	0.033	*0.163*
LU	0.135	0.218	0.053	0.243	0.173	0.054	*0.315*	0.145	0.018	*0.125*	0.183	0.024	*0.131*	0.157	0.033	*0.211*	0.225	0.030	*0.132*	0.198	0.048	*0.243*	0.212	0.029	*0.139*
BE	0.151	0.232	0.071	0.306	0.215	0.067	*0.309*	0.182	0.040	*0.221*	0.208	0.030	*0.145*	0.188	0.044	*0.232*	0.253	0.039	*0.152*	0.220	0.058	*0.262*	0.210	0.042	*0.202*
DE	0.152	0.239	0.064	0.270	0.215	0.061	*0.284*	0.183	0.027	*0.146*	0.203	0.027	*0.134*	0.189	0.026	*0.138*	0.256	0.041	*0.158*	0.219	0.052	*0.238*	0.239	0.043	*0.178*
CY	0.155	0.246	0.065	0.262	0.229	0.066	*0.289*	0.189	0.025	*0.134*	0.203	0.028	*0.136*	0.232	0.040	*0.174*	0.268	0.033	*0.122*	0.232	0.052	*0.224*	0.249	0.049	*0.196*
PL	0.173	0.266	0.081	0.304	0.289	0.084	*0.292*	0.227	0.051	*0.225*	0.238	0.048	*0.202*	0.226	0.041	*0.182*	0.276	0.032	*0.117*	0.254	0.066	*0.259*	0.288	0.052	*0.181*
IE	0.175	0.272	0.078	0.288	0.238	0.065	*0.275*	0.213	0.045	*0.213*	0.236	0.034	*0.144*	0.216	0.045	*0.210*	0.269	0.040	*0.147*	0.250	0.068	*0.273*	0.253	0.046	*0.184*
PT	0.181	0.279	0.084	0.299	0.241	0.070	*0.292*	0.236	0.060	*0.255*	0.250	0.050	*0.202*	0.232	0.046	*0.198*	0.296	0.043	*0.147*	0.272	0.061	*0.223*	0.273	0.062	*0.227*
UK	0.191	0.300	0.082	0.274	0.261	0.074	*0.282*	0.220	0.031	*0.143*	0.256	0.037	*0.146*	0.242	0.054	*0.222*	0.303	0.049	*0.163*	0.267	0.070	*0.261*	0.279	0.049	*0.174*
LT	0.191	0.288	0.095	0.328	0.273	0.085	*0.309*	0.245	0.071	*0.289*	0.277	0.072	*0.260*	0.235	0.039	*0.166*	0.299	0.035	*0.117*	0.269	0.074	*0.275*	0.307	0.060	*0.197*
EE	0.194	0.296	0.091	0.308	0.245	0.075	*0.306*	0.244	0.063	*0.259*	0.276	0.067	*0.244*	0.232	0.052	*0.224*	0.326	0.051	*0.156*	0.283	0.065	*0.231*	0.304	0.071	*0.232*
ES	0.197	0.314	0.081	0.258	0.268	0.074	*0.277*	0.237	0.034	*0.144*	0.262	0.039	*0.149*	0.246	0.046	*0.186*	0.319	0.051	*0.160*	0.288	0.072	*0.249*	0.284	0.056	*0.198*
IT	0.198	0.304	0.093	0.306	0.276	0.086	*0.313*	0.228	0.034	*0.151*	0.257	0.041	*0.161*	0.254	0.062	*0.243*	0.335	0.055	*0.163*	0.273	0.081	*0.296*	0.301	0.066	*0.221*
GR	0.203	0.301	0.104	0.347	0.272	0.096	*0.353*	0.259	0.053	*0.203*	0.269	0.047	*0.175*	0.273	0.081	*0.299*	0.329	0.042	*0.128*	0.285	0.078	*0.273*	0.298	0.070	*0.235*
LV	0.212	0.314	0.110	0.350	0.319	0.112	*0.351*	0.265	0.083	*0.314*	0.308	0.075	*0.245*	0.263	0.030	*0.116*	0.377	0.059	*0.157*	0.299	0.082	*0.274*	0.364	0.094	*0.257*

8. Multidimensional poverty comparisons between the Italian and the Polish regions: an integrated fuzzy approach.

8.1 Fuzzy Depth Indicators

Fuzzy incidence indicators defined under the FR approach overlook the second basic aspect of poverty analysis, namely poverty depth. The necessity of also taking poverty depth into consideration in multidimensional analyses of poverty has been postulated by many researchers (see, for example Shorrocks and Subramanian, 1994). Panek (2010) proposed to extend the IFR approach by incorporating two additional indicators, namely the Fuzzy Monetary Depth indicator (FMD) and the Fuzzy Monetary Supplementary Depth indicator (FSD).

8.1.1 Fuzzy Monetary Depth Indicator (FMD)

The starting point for defining the FMD indicator is the calculation of the income (poverty) gap ratio for each individual:

$$v_i = \frac{z - y_i}{z}, \qquad i=1,2,...,n, \tag{8.1}$$

with the non-poor individuals v_i being assigned the value of zero.

In the next step, we define the degree of the lack of poverty gap (non-poverty gap score) for each individual:

$$d_i = 1 - v_i, \qquad i=1,2,...,n. \tag{8.2}$$

d_i is a positive score indicating a lack of poverty gap and is analogous to y_i in the construction of the FMI indicator.

The FMD indicator is defined, similarly to the FMI indicator, as a combination of the $(1\text{-}F_{(MD)})$ indicator and the $(1\text{-}L_{(MD)})$ indicator.

The $(1\text{-}F_{(MD),i})$ indicator for the i-th individual is the proportion of individuals whose non-poverty gap score is higher (who are not as poor or better off) than the individual concerned:

$$\mu_i(d) = FMD_i = \left(1 - F_{(MD),i}\right)^{\beta} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma}{\sum_{\gamma=2}^{n} w_\gamma} \right)^{\beta}, \; i=1,2,...,n; \qquad \mu_n(d)=0, \tag{8.3}$$

where:

$F_{(MD),i}$ – value of the distribution function $F(d_i)$ of the non-poverty gap score for the *i*-th individual,

w_γ - weight of the *i*-th individual of rank γ in ascending non-poverty gap score distribution,

β - parameter.

The (1-L(MD),i) indicator is the share of the total non-poverty gap score assigned to all individuals whose non-poverty gap score is higher (who are not as poor or are better off) than the individual concerned:

$$\mu_i(d) = FMD_i = \left(1 - L_{(MD),i}\right)^\beta = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma d_\gamma}{\sum_{\gamma=2}^{n} w_\gamma d_\gamma} \right)^\beta, \; i=1,2,...,n; \quad \mu_n(d)=0, \tag{8.4}$$

where:

$L_{(MD),i}$ – value of the Lorenz curve of the non-poverty gap score $L\left(F\left(d_i\right)\right)$ for the *i*-th person.

Finally, the degree of poverty gap, for the *i*-th individual, is defined as a combination of formulas (8.3) and (8.4):

$$\mu_i(d) = FMD_i = \left(1 - F_{(MD),i}\right)^{\beta-1}\left(1 - L_{(MD),i}\right), \qquad i=1,2,...,n. \tag{8.5}$$

The overall (for the population in question) Fuzzy Monetary Depth indicator (the depth of relative deprivation indicator) is calculated as follows:

$$FMD = \frac{\sum_{i=1}^{n} \mu_i(d) \cdot w_i}{\sum_{i=1}^{n} w_i}. \tag{8.6}$$

The parameter β in equation (8.5) is estimated so that the mean of the FMD indicator (for the entire population) is equal to the poverty gap index.

8.1.2 Fuzzy Supplementary Depth indicator (FSD)

The starting point for calculating the FSD indicator is the same set of deprivation symptoms as was established for the FSI indicator. Then we determine a quantitative deprivation gap ratio for each deprivation symptom, and for each individual:

$$x_{hj,i} = \frac{\left(c_{hj} = r-1\right) - \left(e_{hj,i} - 1\right)}{\left(c_{hj} = r-1\right)},\ h=1,2,...,m;\ j=1,2,...,k_n;\ i=1,2,...,n, \qquad (8.7)$$

with the non-deprived individuals, with regard to the *j*-th symptom in the h-dimension, $x_{hj,i}$ being set to zero (for individual, for which rank assumes value $c_{hj,i} \geq r$; $c_{nj}=1,2,...,u$; r≤u),

where:

$c_{hj}=r$ – rank of the *j*-th deprivation symptom category in the *h*-th dimension for which deprivation is not found.

The above formula is identical for dichotomous and polychotomous variables (deprivation symptoms).

In the next step the degree of the lack of deprivation gap (non-deprivation gap score) for each individual is calculated:

$$s_{hj,i} = 1 - x_{hj,i}, \qquad h=1,2,...,m;\ j=1,2,...,k_n;\ i=1,2,...,n. \qquad (8.8)$$

Then we determine the deprivation gap score (assessment of the degree of deprivation gap) for each deprivation symptom:

$$g_{hj,i} = \frac{1 - F\left(s_{hj,i}\right)}{1 - F\left(1\right)}, \qquad h=1,2,...,m;\ j=1,2,...,k_h;\ i=1,2,...,n, \qquad (8.9)$$

where:

F(shj,i) – value of the distribution function of the non-deprivation gap score, regarding the j-th deprivation symptom in the h-th dimension, for the i-th individual.

Using the system of weights, the same that was applied in the calculation of FMI indicator, the non-deprivation gap score for the i-th individual, and for each deprivation dimension separately, is determined:

$$g_{h,i} = \frac{\sum_{j=1}^{k_h} w_{hj}\left(1 - g_{hj,i}\right)}{\sum_{j=1}^{k_h} w_{hj}}, \qquad h=1,2,...,m;\ i=1,2,...,n. \qquad (8.10)$$

In the next step the non-deprivation gap scores (8.10) are aggregated into the overall deprivation gap score indicating lack of deprivation gap, for the each *i*-th person, as the unweighted mean:

$$g_i = \frac{\sum_{h=1}^{m} g_{h,i}}{m}, \qquad i=1,2,...,n. \tag{8.11}$$

The FSD indicator, for the *i*-th individual, is calculated as a combination of the (1-$F_{(SD,i)}$) indicator and the (1-$L_{(SD),i}$) indicator:

$$\mu_i(s) = FSD_i = \left(1 - F_{(SD),i}\right)^{\beta'-1}\left(1 - L_{(SD),i}\right), \qquad i=1,2,...,n. \tag{8.12}$$

The (1-$F_{(SD,i)}$) indicator, for the *i*-th individual, is the proportion of individuals non-deprivation gap score is higher (which are less deprived) than the individual concerned:

$$\mu_i(s) = FSD_i = \left(1 - F_{(SD),i}\right)^{\beta'} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma}{\sum_{\gamma=2}^{n} w_\gamma}\right)^{\beta'}, \; i=1,2,...,n; \; \mu_n(s)=0, \tag{8.13}$$

where:

$F_{(SD),i}$ – value of the distribution function $F(g_i)$ of the lack of the deprivation gap score for the *i*-th individual,

w_γ - weight of the *i*-th individual of rank γ in the ascending lack of the deprivation gap score distribution,

β' - parameter.

The (1-$L_{(SD),i}$) indicator, for the *i*-th individual, is the share of the total non-deprivation gap score assigned to all individuals whose non-deprivation gap score is higher than the individual concerned:

$$\mu_i(s) = FSD_i = \left(1 - L_{(SD),i}\right)^{\beta'} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma g_\gamma}{\sum_{\gamma=2}^{n} w_\gamma g_\gamma}\right)^{\beta'}, \; i=1,2,...,n; \; \mu_n(s)=0, \tag{8.14}$$

where:

$L_{(SD),i}$ – value of the Lorenz curve of the non-deprivation gap score for the *i*-th individual.

Finally the Fuzzy Supplementary Depth indicator (the depth of relative deprivation indicator) for the population is defined as the following mean:

$$FSD = \frac{\sum_{i=1}^{n} \mu_i(s) \cdot w_i}{\sum_{i=1}^{n} w_i}, \qquad (8.15)$$

The parameter β' is estimated so that the FSD indicator for the population is equal to the income gap index.

The FSD indicator for the *i*-th individual and for the each *h*-th deprivation dimension is calculated as follows:

$$\mu_i(s_h) = FSD_{h,i} = \left(1 - F_{(SD)h,i}\right)^{\beta'-1}\left(1 - L_{(SD)h,i}\right),\ h=1,2,...,m;\ i=1,2,...,n. \qquad (8.16)$$

Finally, the Fuzzy Supplementary Depth indicators for each *h*-th deprivation dimension for the population is are defined as:

$$FSD_h = \frac{\sum_{i=1}^{n} \mu_i(s_h) \cdot w_i}{\sum_{i=1}^{n} w_i}, \qquad h=1,2,...,m. \qquad (8.17)$$

8.2 Empirical results

8.2.1 Fuzzy poverty incidence in Poland and in Italy by regions

There were considerable differences in average equivalised disposable incomes between Poland and Italy in 2008. Mean adjusted income in Poland (PPS 7755 per equivalent unit) was 2.2 times less than that in Italy (PPS 17029 per equivalent unit). Differences in average income between compared countries result in greater fuzzy monetary poverty incidence in Poland than in Italy as well as in the Polish regions than in the Italian regions.

The fuzzy poverty incidence indicator reached for Poland more than 44.33 percent while only 12.30 percent for Italy (Table 8.1).

Table 8.1 Fuzzy Incidence Indicators by the Italian and the Polish Regions in 2008.

Regions	Indicator values · 100								
	FMI	**FSI**	**$FSI_{h=1}$**	**$FSI_{h=2}$**	**$FSI_{h=3}$**	**$FSI_{h=4}$**	**$FSI_{h=5}$**	**$FSI_{h=6}$**	**$FSI_{h=7}$**
Italy:	**12,30**	**23,43**	**23,81**	**7,22**	**13,77**	**15,06**	**23,24**	**22,28**	**23,52**
Piemonte	8,17	22,53	18,01	6,63	13,43	14,24	28,50	21,66	20,80
Valle d'Aosta	6,57	13,83	12,35	4,73	12,63	11,60	11,27	26,63	17,75
Liguria	10,02	16,91	19,89	8,02	8,68	10,45	20,36	19,28	15,77
Lombardia	7,48	17,96	14,87	5,60	10,15	12,00	24,19	23,63	18,73
Trentino	6,10	15,10	14,88	3,84	10,96	8,03	16,41	23,54	17,80
Alto Adige	5,32	13,26	13,89	3,64	12,40	6,73	15,14	23,50	17,01
Veneto	8,17	19,08	19,84	3,66	13,47	11,50	21,14	24,27	21,13
Friuli-Venezia Giulia	7,80	16,64	17,75	3,58	13,86	11,80	16,78	22,53	19,90
Emilia Romagna	7,29	19,26	16,11	4,51	14,72	11,55	24,67	22,18	20,73
Toscana	7,75	19,52	18,65	4,31	12,10	12,91	21,24	22,81	20,85
Umbria	9,86	19,16	20,05	4,58	13,50	14,98	18,17	20,90	22,92
Marche	9,38	19,65	21,91	4,94	14,20	13,65	18,28	24,60	23,63
Lazio	11,00	25,50	23,44	6,18	12,99	15,30	29,19	18,12	24,81
Abruzzo	12,43	19,89	26,12	4,40	12,14	13,87	10,13	20,36	28,63
Molise	16,80	16,42	23,82	7,42	11,67	11,49	6,38	21,85	25,29
Campania	22,29	35,20	36,65	12,48	18,05	21,50	34,92	22,76	26,81
Puglia	17,19	29,58	35,63	12,28	13,58	19,70	18,78	24,17	28,84
Basilicata	18,09	26,99	32,16	10,37	18,07	18,92	12,81	19,64	31,18
Calabria	21,70	29,23	33,48	11,03	18,53	20,91	14,38	20,10	34,66
Sicilia	21,95	33,79	40,59	11,99	18,12	22,82	21,55	22,11	31,02
Sardegna	14,16	28,70	33,16	10,80	18,53	16,69	16,30	23,49	31,31
Poland:	**44,33**	**26,63**	**25,36**	**9,37**	**14,35**	**26,42**	**19,83**	**19,10**	**25,92**
Dolnośląskie	40,55	29,22	26,96	9,00	15,22	29,39	22,44	18,65	26,35
Kujawsko-pomorskie	47,78	25,65	24,70	12,13	13,99	27,17	18,22	19,25	25,19
Lubelskie	52,82	26,58	26,08	9,67	15,73	26,31	18,39	20,83	24,72
Lubuskie	46,30	27,91	27,91	10,32	14,96	25,15	22,57	18,25	26,01
Łódzkie	47,61	27,52	26,91	10,16	15,34	26,96	18,74	19,58	26,99
Małopolskie	43,38	27,18	24,43	8,98	14,78	26,05	21,01	17,79	26,80
Mazowieckie	38,54	25,91	24,35	9,66	13,40	26,42	19,86	17,91	26,11
Opolskie	39,82	26,67	24,49	9,65	11,09	23,25	21,75	18,09	24,87
Podkarpackie	53,07	26,15	24,87	8,45	13,48	26,40	19,45	18,86	25,83
Podlaskie	47,79	25,51	23,93	7,31	12,31	27,70	17,69	20,50	26,82
Pomorskie	41,88	28,80	25,33	9,76	14,68	28,61	22,84	20,71	26,11
Śląskie	40,05	26,56	26,08	9,41	15,38	26,41	19,59	20,20	25,78
Świętokrzyskie	50,69	24,99	24,59	8,86	14,12	23,30	18,65	16,34	24,88
Warmińsko-mazurskie	51,26	23,62	23,10	8,49	12,67	24,60	17,14	19,27	24,03
Wielkopolskie	45,46	26,67	25,20	9,20	14,07	25,99	19,98	18,75	27,03
Zachodniopomorskie	40,56	24,79	25,90	7,47	14,67	23,73	17,41	20,51	24,07

*Coefficient of variation in percentages.

Source: Central Statistical Office, EU-SILC Survey data, wave 4. Survey co-financed by UE. The views expressed are solely those of the author and should not be attributed to the European Commission.

The lowest value of fuzzy poverty incidence indicator in the Polish region (Mazowieckie) was 1.73 times higher than the highest value of that indicator in the Italian region (Campania). Regions of the greatest fuzzy poverty incidence were in 2008 Podkarpackie, Lubelskie and Warmińsko-Mazurskie and those with the lowest were Alto Adige, Trentino and Valle d'Aosta.

The fuzzy poverty threat in Poland is considerably higher than the fuzzy deprivation threat. An opposite situation in Italy is observed. In Poland the fuzzy deprivation indicators have lower values than in Italy only in the environmental problems dimension and in the education and labour market dimension. The greatest overall FSI indicator values were observed in Campania, Sicilia, Puglia, Calabria and Dolnośląskie while the lowest were noted in Alto Adige and Valle d'Aosta. Regions of the greatest fuzzy deprivation incidence in the basic life style dimension were Sicilia, Campania and Puglia and those of the lowest were Valle d'Aosta, Alto Adige, Lombardia and Trentino. The highest level of fuzzy deprivation incidence in the equipment of households in durables was observed in Campania, Puglia and Kujawsko-Pomorskie and the lowest in Friuli-Venezia Giulia, Alto Adige, Veneto and Trentino.

Regions noted the greatest fuzzy deprivation incidence in the housing facilities and deterioration dimension were Calabria, Sardegna, Sicilia, Basilicata and Campania and the lowest were Liguria, Lombardia and Trentino. When it comes to the household arrears and unexpected financial expenses dimension, the worst situation was in Dolnośląskie, Pomorskie, Podlaskie and Kujawsko-Pomorskie, and the best situation was in Alto Adige, Trentino and Liguria. Campania, Lazio and Piemonte had the highest values of the FSI indicator in the environmental problems dimension. In contrast Molise, Abruzzo and Valle d'Aosta had the lowest values of that indicator.

Among the regions, the poverty deprivation threat in the education and labour market dimension was the highest in Valle d'Aosta, Marche, Veneto and Puglia, while the lowest in Świętokrzyskie, Małopolskie and Mazowieckie.

Regarding the health dimension, the worst situation was in Calabria, Sardegna, Basilicata and Sicilia, while the best in Liguria, Alto Adige, Valle d'Aosta and Trentino.

8.2.2 Fuzzy poverty depth in Poland and in Italy by regions

The fuzzy poverty depth noted in Poland as a whole in 2008 was much higher than in Italy(Table 8.2).

Table 8.2. Fuzzy Depth Indicators by the Italian and the Polish Regions in 2008.

Regions	Indicator values · 100								
	FMD	**FSD**	**$FSD_{h=1}$**	**$FSD_{h=2}$**	**$FSD_{h=3}$**	**$FSD_{h=4}$**	**$FSD_{h=5}$**	**$FSD_{h=6}$**	**$FSD_{h=7}$**
Italy:	**2,96**	**5,97**	**8,06**	**3,41**	**4,46**	**6,41**	**12,09**	**9,13**	**8,39**
Piemonte	1,37	5,65	4,90	2,87	4,47	6,60	17,96	9,04	7,31
Valle d'Aosta	0,46	2,11	2,46	1,99	4,68	4,37	5,23	12,32	4,70
Liguria	2,43	3,03	5,68	3,61	2,97	3,24	11,30	8,14	3,38
Lombardia	1,72	3,31	3,67	2,67	3,09	4,81	12,14	10,53	5,22
Trentino	1,17	2,20	4,07	1,90	4,47	3,01	6,39	11,23	5,93
Alto Adige	0,02	1,93	4,14	0,98	3,55	1,77	5,49	10,25	3,34
Veneto	0,14	3,20	5,26	1,58	3,43	4,29	9,31	10,20	6,62
Friuli-Venezia Giulia	0,14	2,67	4,64	1,56	3,92	3,99	6,95	9,63	5,42
Emilia Romagna	1,62	3,77	4,54	1,78	4,35	4,91	12,65	9,53	6,19
Toscana	1,62	4,49	5,61	1,89	4,12	5,57	10,85	9,82	6,49
Umbria	1,85	3,21	4,70	2,19	3,67	6,18	7,94	8,78	6,45
Marche	2,17	3,81	5,54	2,08	4,41	5,04	6,92	10,12	7,62
Lazio	2,56	5,73	7,15	2,87	4,28	6,51	16,83	7,41	9,01
Abruzzo	0,34	3,63	6,58	2,25	3,81	6,23	3,87	7,17	11,07
Molise	3,56	2,03	4,64	3,20	2,69	3,50	1,65	8,83	8,97
Campania	6,86	12,70	15,91	6,56	7,05	10,59	22,11	8,02	10,20
Puglia	4,26	9,52	15,35	6,07	4,66	8,23	8,74	9,00	12,35
Basilicata	4,12	7,71	12,89	5,95	5,78	8,40	4,96	6,90	13,09
Calabria	5,68	7,84	10,56	5,62	6,48	8,78	5,04	7,03	17,16
Sicilia	5,58	11,84	17,54	5,65	6,18	10,64	9,67	8,60	14,06
Sardegna	3,02	8,74	13,84	4,87	5,43	5,64	7,28	9,21	13,27
Poland:	**15,91**	**13,12**	**18,45**	**11,26**	**9,98**	**12,44**	**7,19**	**3,29**	**12,06**
Dolnośląskie	13,89	15,35	19,13	11,47	11,65	15,16	11,24	3,14	12,87
Kujawsko-pomorskie	17,03	10,27	13,14	10,51	9,73	10,44	8,27	4,51	8,71
Lubelskie	23,66	15,00	19,90	14,34	15,32	10,35	4,60	4,27	14,84
Lubuskie	17,17	14,66	25,59	13,14	8,44	11,37	6,30	2,39	12,50
Łódzkie	17,56	15,80	20,44	13,29	14,55	14,13	7,82	2,99	11,82
Małopolskie	14,74	16,22	21,92	12,43	9,30	15,82	7,95	3,35	14,55
Mazowieckie	12,74	11,93	14,48	11,85	10,04	11,64	7,05	2,88	13,30
Opolskie	11,56	8,91	18,25	9,37	4,91	8,63	5,66	4,50	9,58
Podkarpackie	21,76	12,76	22,10	10,96	8,60	12,49	3,68	3,40	12,77
Podlaskie	17,81	10,49	13,22	9,04	11,08	9,88	3,75	3,57	14,33
Pomorskie	14,76	11,23	14,97	9,89	7,80	11,43	8,74	3,89	8,83
Śląskie	12,99	12,53	19,05	9,41	8,30	12,37	9,30	2,65	10,17
Świętokrzyskie	20,67	17,59	28,00	12,22	14,37	16,23	3,37	2,56	15,18
Warmińsko-mazurskie	19,03	13,75	21,02	10,55	9,99	10,73	5,03	3,34	13,89
Wielkopolskie	15,84	10,37	14,43	10,15	8,50	11,57	6,20	2,85	9,14
Zachodniopomorskie	14,10	13,40	21,96	11,64	6,35	12,66	5,52	4,64	13,13

*Coefficient of variation in percentages.

Source: Central Statistical Office, EU-SILC Survey data, wave 4. Survey co-financed by UE. The views expressed are solely those of the author and should not be attributed to the European Commission.

The same applies to the Polish and the Italian regions. The hierarchy of regions according to the fuzzy poverty depth is different from the hierarchy observed in the hierarchy of regions regarding the fuzzy poverty incidence. The highest fuzzy poverty depth was noted in Lubuskie, Podkarpackie and Świętokrzyskie. The lowest fuzzy poverty depth was found in Alto-Adige, Veneto and Friuli-Venezia Giulia.

The value of the FMD indicator was Poland is higher than the most values of the FSD indicators except the FSD indicator value for the basic life style dimension. An opposite situation in Italy is seen. The FPD indicator value was lower than FSD indicator values for all the deprivation dimensions. Moreover, the FPD indicators in Italy had greater values than in Poland except the environmental problems dimension and the education and labour market dimension. There were considerable differences in FSD indicators values between regions for all deprivation dimensions. The highest level of fuzzy deprivation depth, for all deprivation dimension together, was noted in Świętokrzyskie and Małopolskie, and the lowest in Alto Adige, Friuli-Venezia Giulia and Liguria.

Regions noted the greatest fuzzy deprivation depth in the basic life style dimension were Świętokrzyskie, Lubuskie and Podkarpackie and the lowest were Valle d'Aosta and Lombardia.

The worst situation in the equipment of households in durables is found in Lubelskie, Łódzkie and Lubuskie, while the best situation is observed in Alto Adige, Friuli-Venezia Giulia and Veneto.

According to the housing facilities and deterioration dimension the worst situation was observe in Lubelskie, Łódzkie and Świętokrzyskie, and the best in Molise and Liguria.

Looking at the household arrears and unexpected financial expenses we can observe the greatest FSD indicator values in Świętokrzyskie, Dolnośląskie and Łódzkie, while the lowest in Alto Adige, Trentino and Liguria.

In Campania, Piemonte and Lazio the FSD indicator assumed notably higher values in the environmental problems dimension than in the other regions. That indicator values were the lowest in Molise, Świętokrzyskie, Podkarpackie, Podlaskie and Abruzzo. Valle d'Aosta and Trentino had the highest FSD indicator values in the

education and labour market dimension while the lowest values were noted in Lubuskie, Świętokrzyskie and Śląskie.

The highest FSD indicator values for the health dimension were noted in Świętokrzyskie, Lubelskie, Małopolskie, Podlaskie and Sicilia, and the lowest in Alto Adige, Liguria and Valle d'Aosta.

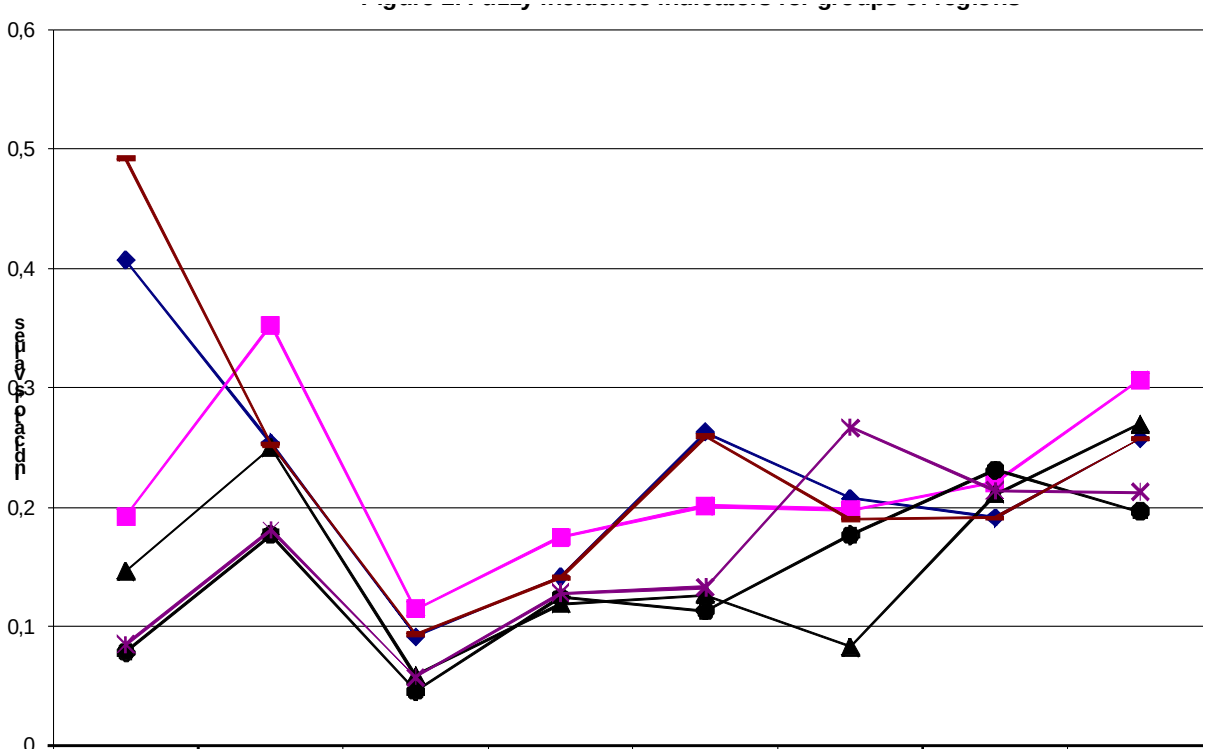
8.2.3 Grouping regions regarding to similarity of fuzzy poverty incidence structure

Using the *k*-means method the following groups of regions, according to the similarity of the structure of fuzzy incidence indicators, were obtained:

- group 1: Dolnośląskie, Małopolskie, Mazowieckie, Opolskie, Pomorskie, Śląskie, Zachodniopomorskie.
- group 2: Campania, Puglia, Basilicata, Calabria, Sicilia, Sardegna.
- group 3: Ambruzzo, Molise.
- group 4: Valle d'Aosta, Liguria, Trentino, Alto Adige, Veneto, Friuli-Venezia Giulia, Toscana, Umbria, Marche.
- group 5: Piemonte, Lombardia, Emilia Romagna, Lazio.
- group 6: Kujawsko-Pomorskie, Lubelskie, Lubuskie, Łódzkie, Podkarpackie, Podlaskie, Świętokrzyskie, Warmińsko-Mazurskie, Wielkopolskie.

The Polish regions and the Italian regions are in different groups. It shows that the structure of the fuzzy incidence indicators in the Polish regions and in the Italian regions is very difference. The Polish regions belong to two groups: group 6 and group 1. The group 6 is distinguished by the highest mean value of fuzzy incidence indicator in the monetary dimension and the lowest mean value of fuzzy incidence indicator in the education and labour market dimension (Figure 8.1).

Figure 8.1 Fuzzy incidence indicators for groups of regions.



That group is also characterized by almost the highest poverty threat (the second place in the ranking) in the equipment of household in durables dimension, in the housing facilities and deterioration dimension and in the arrears on mortgage or rent payment dimension.

In the group 1 the highest mean values of fuzzy incidence indicator in the household arrears and unexpected financial expenses dimension are noted. Moreover almost the highest values (the second place in the ranking) of fuzzy incidence indicator in the monetary dimension, in the basic life style dimension, in the housing facilities and deterioration dimension and in the equipment of household in durables dimension are observed.

The Italian regions are divided into four homogeneous groups. Group 2 comprises the Italian regions with the worst situation in the most of the deprivation dimensions. This group has the highest mean values of fuzzy incidence indicators in the basic style dimension, in the housing facilities and deterioration dimension and in the health dimension.

Group 4 encompasses the Italian regions with the best situation in the most of the poverty dimensions. This group has the lowest mean value of the fuzzy incidence indicator in the monetary dimension, in the basic life style dimension, in the equipment of household durables dimension, in the household and unexpected financial expenses dimension and in the health dimension. This is the contrast to the highest poverty threat that was seen in the labour market dimension. In the group 5 the relatively low poverty threat, in the most of poverty dimensions, was observed. An exception is the highest value of the fuzzy incidence indicator in the environmental problem dimension. The group 3 is the group with the lowest mean value of the fuzzy incidence indicator in the housing facilities and deterioration dimension. This group is classified in the middle of the ranking regarding to poverty incidence threat in the other poverty dimensions.

8.2.4 Grouping regions regarding to similarity of fuzzy poverty depth structure

Applying the *k*-means method regions were classified into the following groups regarding to similarity of the structure of fuzzy depth indicators:

Group 1: Lubelskie, Lubuskie, Łódzkie, Małopolskie, Podkarpackie, Świętokrzyskie, Warmińsko-mazurskie, Zachodniopomorskie.

Group 2: Dolnośląskie, Kujawsko-pomorskie, Mazowieckie, Opolskie, Podlaskie, Pomorskie, Śląskie, Wielkopolskie.

Group 3: Puglia, Basilicata, Calabria, Sicilia, Sardegna.

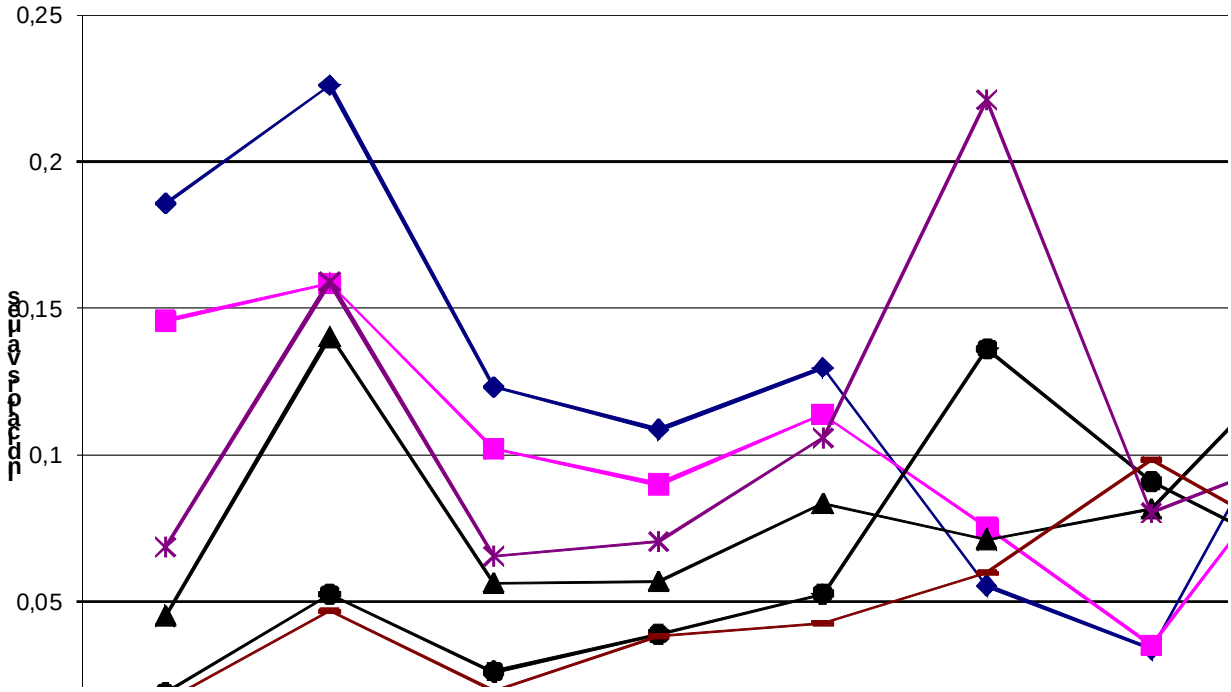
Group 4: Piemonte, Liguria, Lombardia, Emilia Romana, Toscana, Lazio.

Group 5: Campania.

Group 6: Valle d'Aosta, Trentino, Alto Adige, Veneto, Friuli-Venezia, Giulia, Umbria, Marche, Abruzzo, Molise.

The Polish regions and the Italian regions are in different groups. Moreover, the groups composition is not the same as in the regions classification regarding to similarity of fuzzy poverty incidence structure. The Polish regions were divided into two groups: group 1 and group 2. The group 1 is characterized by the worst situation in the monetary dimension and in the all financial deprivation dimensions expressed in non-monetary variables (the four first dimensions). At the same time the lowest deprivation threat in the environmental problems dimension and in the education and labour market dimension were noted (Figure 8.2).

Figure 8.2. Fuzzy depth indicators for groups of regions



Group 2 is classified on the second place of the ranking regarding to the worst situation in the poverty dimension and in the financial deprivation dimensions. An exception is its position in the ranking according to the equipment of households in durables dimension. The Italian regions belong to four groups: 4, 5, and 6. The best situation regarding to the poverty depth threat is found in group 6. An exception is the highest value of the fuzzy deprivation indicator in the education and labour market dimension. A bit worse situation than in group 6 is observed in group 4. This group is also characterized by the lowest deprivation depth threat in the health dimension.

Group 3 is classified in the middle of ranking regarding to poverty depth threat in the most of poverty dimensions. This group has, however the highest fuzzy depth indicator value in the health dimension.

Group 5 encompasses only Campania. This one-element group has the highest poverty depth threat among the groups of the Italian regions in almost all of the poverty dimensions. Moreover this group has the highest fuzzy depth indicator value among all of the regions groups in the environmental problems dimension.

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